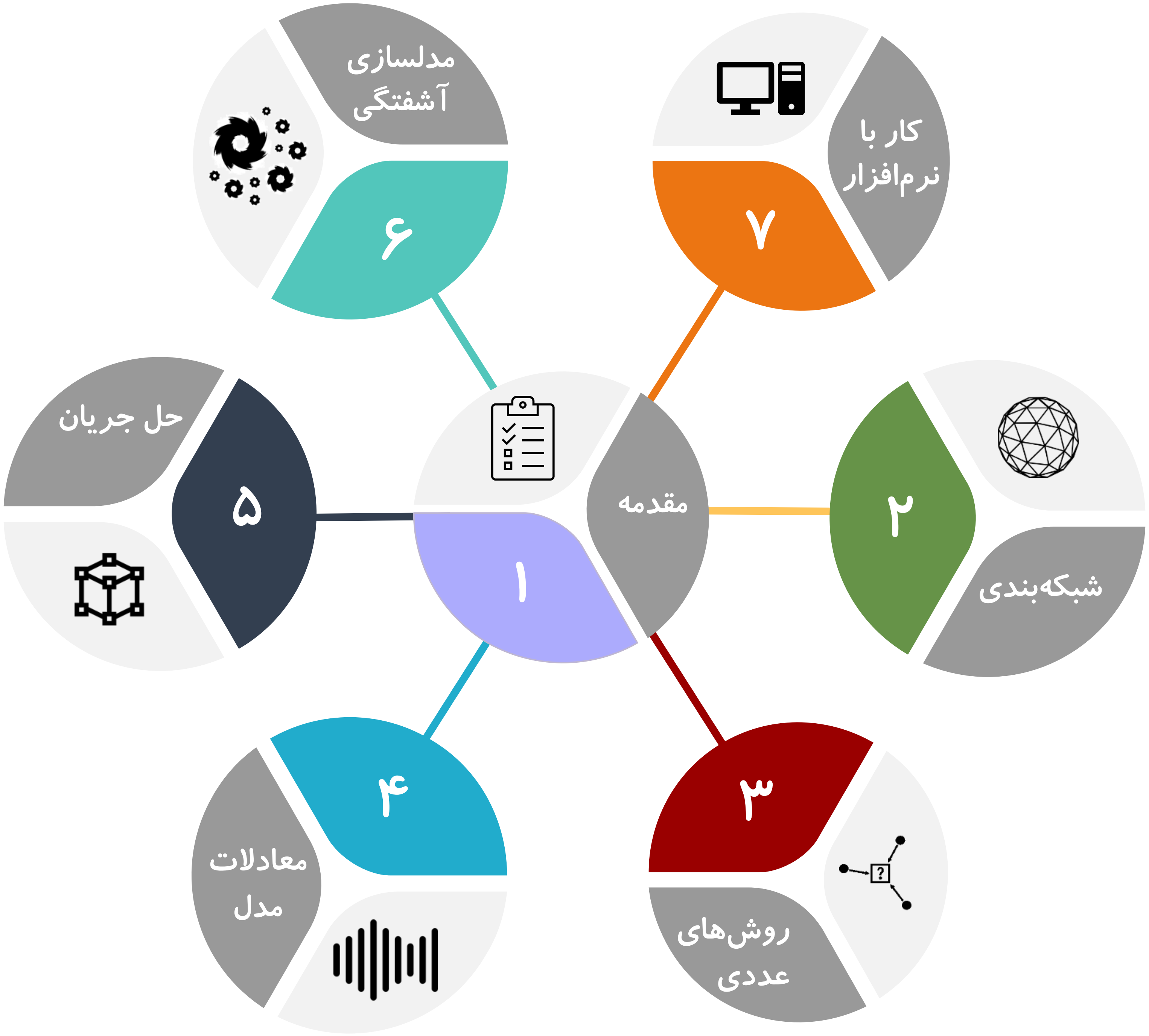
بسم الله الرحمن الرحیم

مقدمات سیالات محاسباتی

محمدصادق کریمی

دانشکده مهندسی مکانیک، دانشگاه صنعتی شریف

پاییز ۱۴۰۰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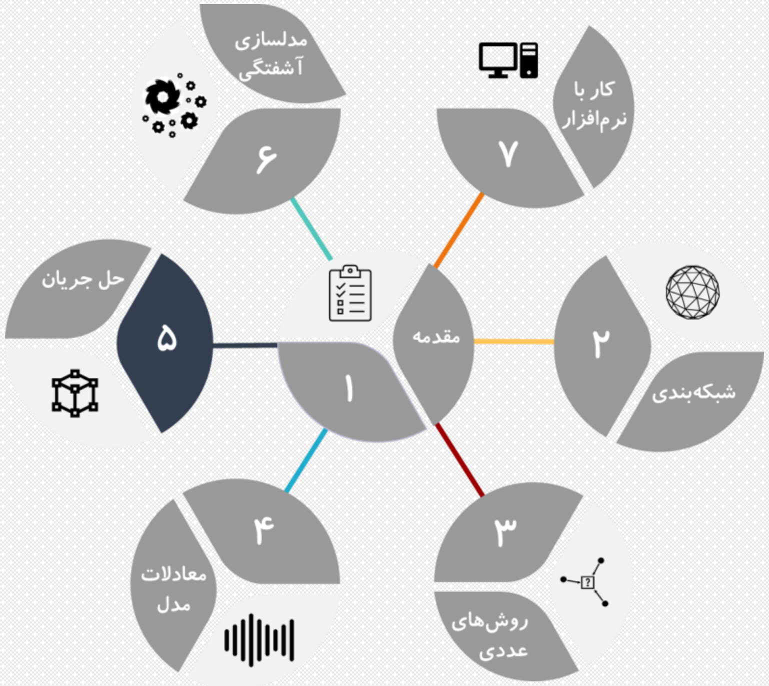

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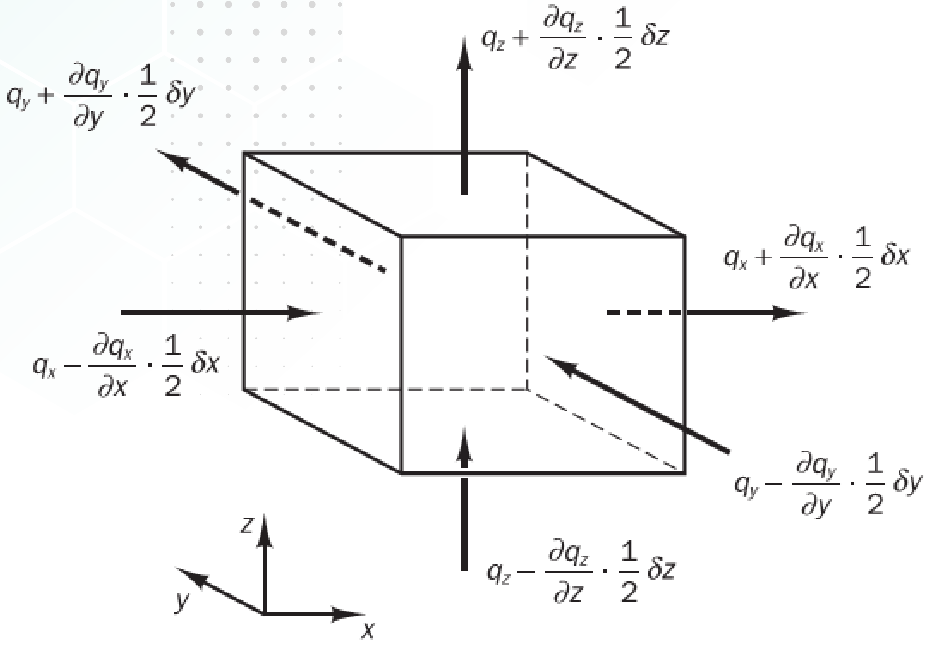
معادله انرژی



نرخ افزایش انرژی ذره سیال	=	نرخ خالص حرارت اضافه شده به ذره سیال	+	نرخ خالص کار انجام شده روی ذره سیال

$$\rho\frac{\mathrm{D}i}{\mathrm{D}t} = -p\nabla.\vec{V} + \nabla.(k\nabla T) + \Phi + S_i$$

$$h = i + \frac{p}{\rho} \qquad \rho\left(\frac{\partial h}{\partial t} + \nabla.(h\vec{V})\right) = -\frac{\mathrm{D}p}{\mathrm{D}t} + \nabla.(k\nabla T) + \Phi + S_i$$

$$\Phi = (\bar{\bar{\tau}}.\nabla)\vec{V} = \tau_{ij}\frac{\partial V_i}{\partial x_j} \qquad \tau_{ij} = \mu\left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i}\right) + \lambda(\nabla\cdot\vec{V})\delta_{ij} = \mu\left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} - \frac{2}{3}(\nabla\cdot\vec{V})\delta_{ij}\right)$$

$$p = p(\rho, t) = \rho\mathrm{R}T \qquad h = h(\rho, T) = c_p T$$

معادله انرژی

سیال تراکم‌ناپذیر در دو بعد

در سیال تراکم‌ناپذیر معادله انرژی به صورت مستقل قابل حل است.

$$\rho c_p\left(\frac{\partial T}{\partial t}+(\vec{V}.\nabla)T\right)=k\nabla^2 T+\Phi$$

$$\rho c_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\Phi$$

$$\Phi=\mu\left(2\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right)$$

$$t^*=\frac{t}{L/U_\infty},x^*=\frac{x}{L},y^*=\frac{y}{L},u^*=\frac{u}{U_\infty},v^*=\frac{v}{U_\infty},T^*=\frac{T-T_\infty}{\Delta T}$$

$$\frac{\partial T^*}{\partial t^*}+\underbrace{u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}}_{\text{Convective}}=\underbrace{\frac{1}{\mathrm{Re.Pr}}\left(\frac{\partial^2 T^*}{\partial x^{*2}}+\frac{\partial^2 T^*}{\partial y^{*2}}\right)}_{\text{Diffusive (Conduction)}}+\underbrace{\frac{\mathrm{Ec}}{\mathrm{Re}}\Phi^*}_{\text{Viscous Dissipation}}$$

$$\Phi^*=2\left(\frac{\partial u^*}{\partial x^*}\right)^2+2\left(\frac{\partial v^*}{\partial y^*}\right)^2+\left(\frac{\partial v^*}{\partial x^*}+\frac{\partial u^*}{\partial y^*}\right)^2$$

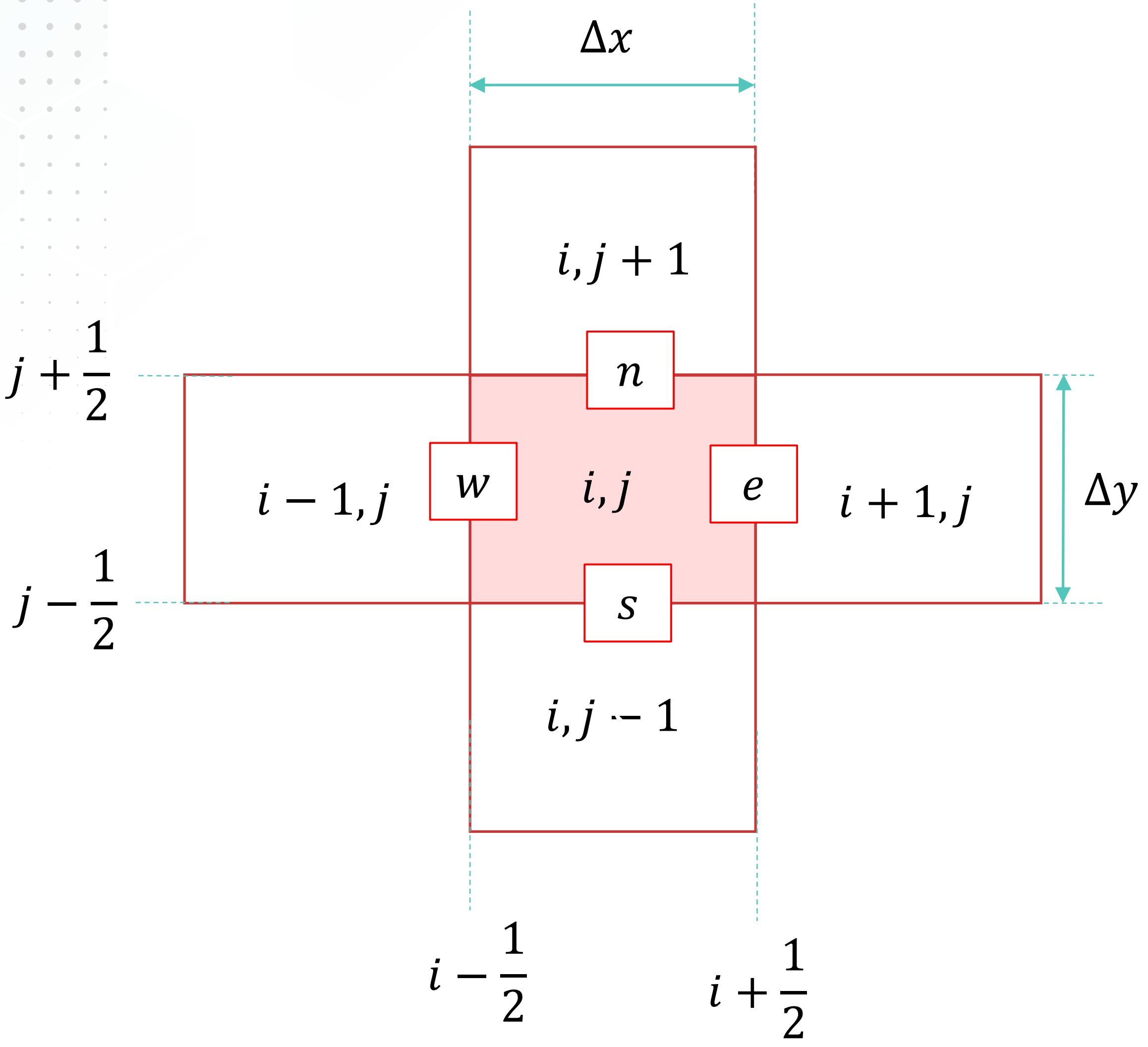
$$\mathrm{Re}=\frac{\rho U_\infty L}{\mu},\mathrm{Pr}=\frac{\nu}{\alpha},\mathrm{Ec}=\frac{U_\infty^2}{c_p\Delta T}$$

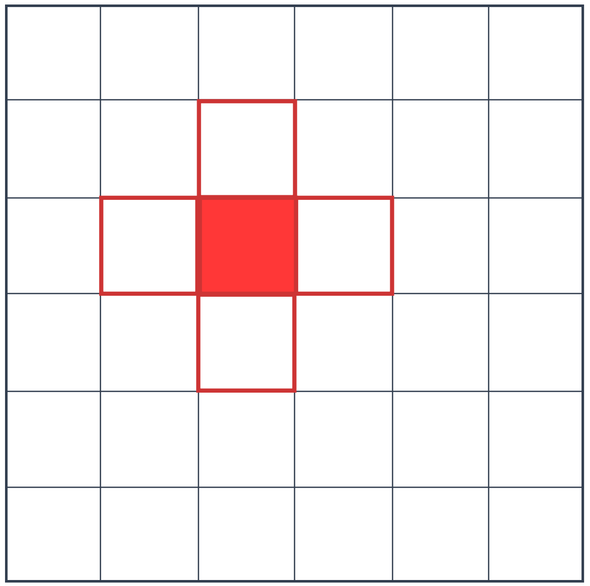
$$\nu=\frac{\mu}{\rho},\alpha=\frac{k}{\rho c_p}$$

معادله انرژی

حجم محدود

تولید شبکه





معادله انرژی

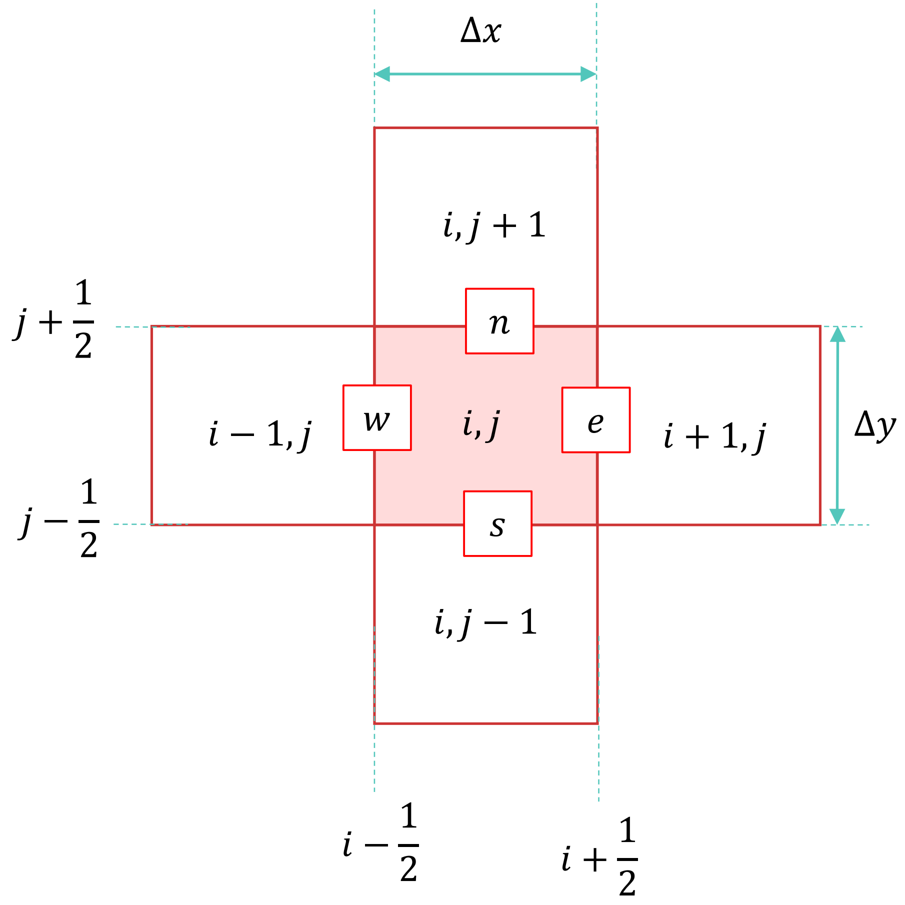
حجم محدود

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\mathrm{Re.\,Pr}}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+S$$

$$\frac{\partial T}{\partial t}+\nabla.(\vec{V}T)-\frac{1}{\mathrm{Re.\,Pr}}\nabla.(\nabla T)-S=0$$

$$\frac{1}{A_{i,j}}\int_{A_{i,j}}\left(\frac{\partial T}{\partial t}+\nabla.(\vec{V}T)-\frac{1}{\mathrm{Re.\,Pr}}\nabla.(\nabla T)-S\right)\mathrm{d}A=$$

$$\frac{1}{A_{i,j}}\int_{A_{i,j}}\left(\frac{\partial T}{\partial t}+\nabla.\begin{bmatrix}uT\\ vT\end{bmatrix}-\frac{1}{\mathrm{Re.\,Pr}}\nabla.\begin{bmatrix}\frac{\partial T}{\partial x}\\ \frac{\partial T}{\partial y}\end{bmatrix}-S\right)\mathrm{d}A=0$$



معادله انرژی

حجم محدود

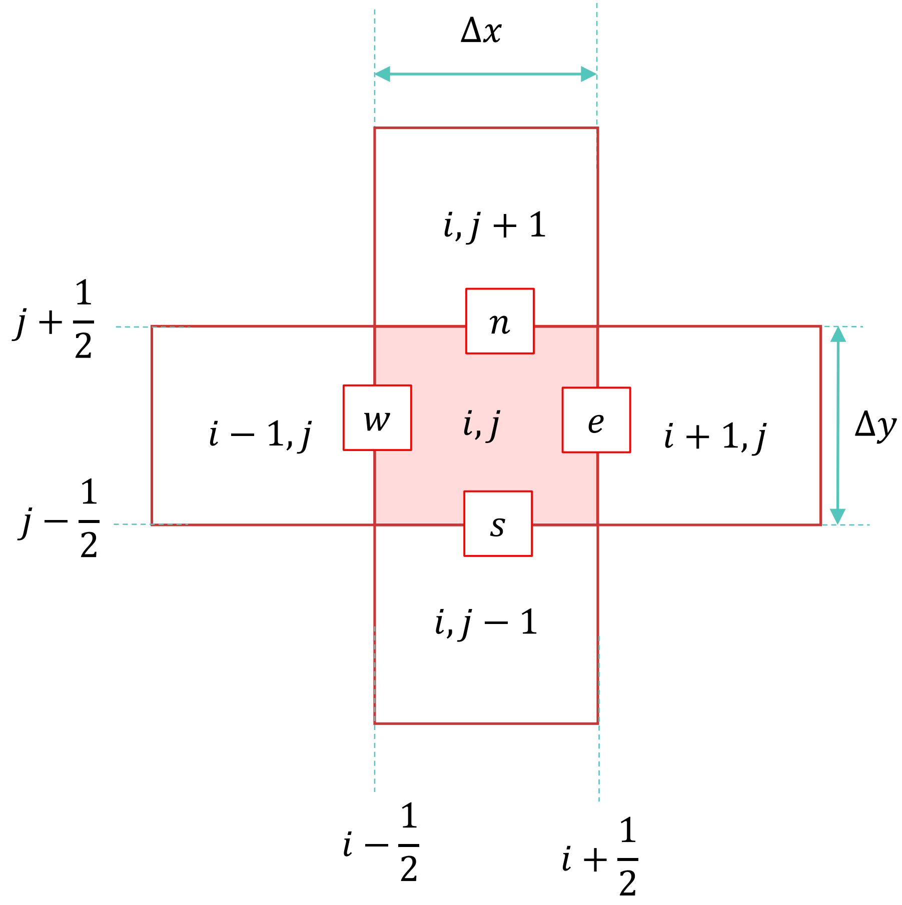
$$\frac{1}{A_{i,j}}\int_{A_{i,j}}\left(\frac{\partial T}{\partial t}\right)\mathrm{d}A=\frac{\partial \bar{T}_{i,j}}{\partial t}$$

$$\frac{1}{A_{i,j}}\int_{A_{i,j}}\left(\nabla.\begin{bmatrix}uT\\vT\end{bmatrix}\right)\mathrm{d}A=\frac{1}{A_{i,j}}\oint_{\partial A_{i,j}}\begin{bmatrix}uT\\vT\end{bmatrix}.\hat{n}\,\mathrm{d}l$$

$$\frac{1}{A_{i,j}}\int_{A_{i,j}}\left(\frac{1}{\mathrm{Re.Pr}}\nabla.\begin{bmatrix}\frac{\partial T}{\partial x}\\\frac{\partial T}{\partial y}\end{bmatrix}\right)\mathrm{d}A=\frac{1}{\mathrm{Re.Pr}}\frac{1}{A_{i,j}}\oint_{\partial A_{i,j}}\begin{bmatrix}\frac{\partial T}{\partial x}\\\frac{\partial T}{\partial y}\end{bmatrix}.\hat{n}\,\mathrm{d}l$$

$$\frac{1}{A_{i,j}}\int_{A_{i,j}}(S)\mathrm{d}A=\bar{S}_{i,j}\cong S_{c_{i,j}}$$

$$\frac{\partial \bar{T}_{i,j}}{\partial t}+\frac{1}{A_{i,j}}\oint_{\partial A_{i,j}}\begin{bmatrix}uT\\vT\end{bmatrix}.\hat{n}\,\mathrm{d}l-\frac{1}{\mathrm{Re.Pr}}\frac{1}{A_{i,j}}\oint_{\partial A_{i,j}}\begin{bmatrix}\frac{\partial T}{\partial x}\\\frac{\partial T}{\partial y}\end{bmatrix}.\hat{n}\,\mathrm{d}l-\bar{S}_{i,j}=0$$



حجم محدود

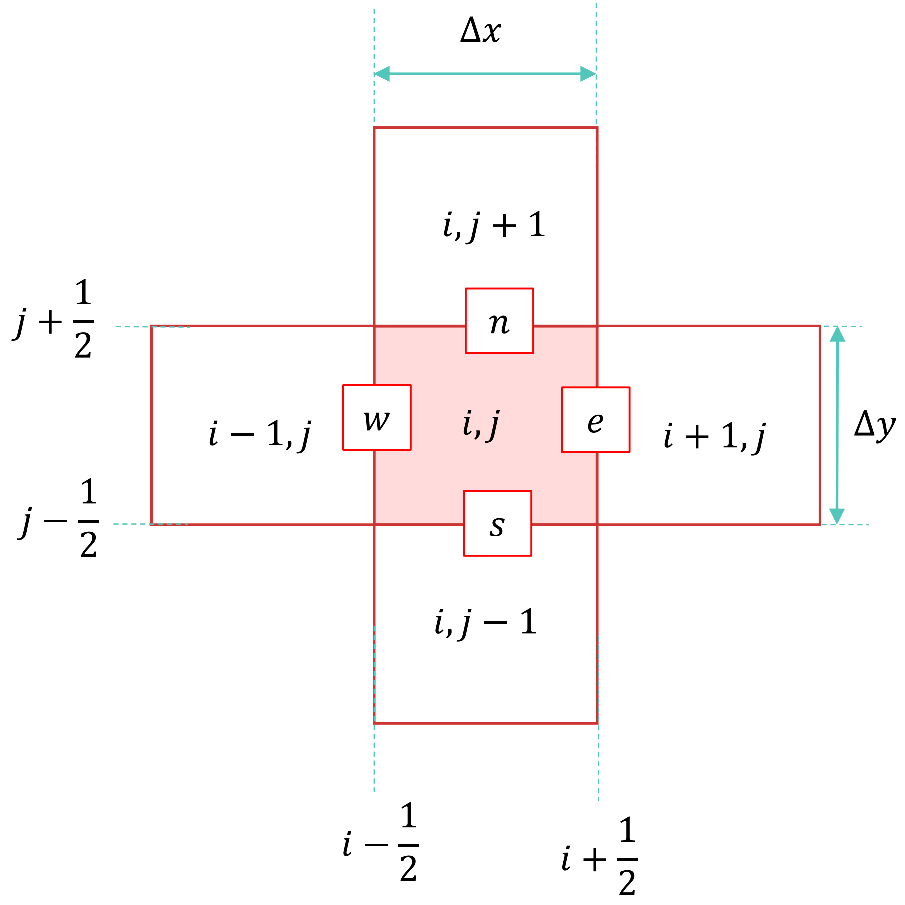
$$\oint_{\partial A_{i,j}} \vec{V}T \,.\, \hat{n}\, \mathrm{d}l =$$

$$\int_s \left(\vec{V}T\Big|_s .\, (-\hat{j})\right) \mathrm{d}x \;+\int_e \left(\vec{V}T\Big|_e .\, (\hat{\imath})\right) \mathrm{d}y$$

$$+\int_n \left(\vec{V}T\Big|_n .\, (\hat{j})\right) (-\mathrm{d}x) \;+\int_w \left(\vec{V}T\Big|_w .\, (-\hat{\imath})\right) (-\mathrm{d}y) =$$

$$\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} \left(\begin{bmatrix} uT \\ vT \end{bmatrix}_s \cdot \begin{bmatrix} 0 \\ -1 \end{bmatrix}\right) \mathrm{d}x \;+\int_{y_{j-\frac{1}{2}}}^{y_{j+\frac{1}{2}}} \left(\begin{bmatrix} uT \\ vT \end{bmatrix}_e \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) \mathrm{d}y$$

$$+\int_{x_{i+\frac{1}{2}}}^{x_{i-\frac{1}{2}}} \left(\begin{bmatrix} uT \\ vT \end{bmatrix}_n \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) (-\mathrm{d}x) \;+\int_{y_{j+\frac{1}{2}}}^{y_{j-\frac{1}{2}}} \left(\begin{bmatrix} uT \\ vT \end{bmatrix}_w \cdot \begin{bmatrix} -1 \\ 0 \end{bmatrix}\right) (-\mathrm{d}y) =$$



$$\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}}\left(\begin{bmatrix}uT\\vT\end{bmatrix}_s\cdot\begin{bmatrix}0\\-1\end{bmatrix}\right)\mathrm{d}x+\int_{y_{j-\frac{1}{2}}}^{y_{j+\frac{1}{2}}}\left(\begin{bmatrix}uT\\vT\end{bmatrix}_e\cdot\begin{bmatrix}1\\0\end{bmatrix}\right)\mathrm{d}y+\int_{x_{i+\frac{1}{2}}}^{x_{i-\frac{1}{2}}}\left(\begin{bmatrix}uT\\vT\end{bmatrix}_n\cdot\begin{bmatrix}0\\1\end{bmatrix}\right)(-\mathrm{d}x)+\int_{y_{j+\frac{1}{2}}}^{y_{j-\frac{1}{2}}}\left(\begin{bmatrix}uT\\vT\end{bmatrix}_w\cdot\begin{bmatrix}-1\\0\end{bmatrix}\right)(-\mathrm{d}y)=$$

$$-\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}}vT\Big|_s\,\mathrm{d}x+\int_{y_{j-\frac{1}{2}}}^{y_{j-\frac{1}{2}}}uT\Big|_e\,\mathrm{d}y+\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}}vT\Big|_n\,\mathrm{d}x-\int_{y_{j-\frac{1}{2}}}^{y_{j-\frac{1}{2}}}uT\Big|_w\,\mathrm{d}y$$

$$vT\Big|_s:\qquad vT\Big|_s=vT(x,y)\Big|_{y=y_{j-\frac{1}{2}}}=vT(x,y)\Big|_{x=x_i,y=y_{j-\frac{1}{2}}}+\frac{\partial}{\partial x}vT(x,y)\Big|_{x=x_i,y=y_{j-\frac{1}{2}}}(x-x_i)+\cdots$$

$$\oint_{\partial A_{i,j}}\vec{V}T\,.\,\hat{n}\,\mathrm{d}l\approx-\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}}vT\Big|_{x=x_i,y=y_{j-\frac{1}{2}}}\mathrm{d}x+\int_{y_{j-\frac{1}{2}}}^{y_{j-\frac{1}{2}}}uT\Big|_{e,c}\mathrm{d}y+\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}}vT\Big|_{n,c}\mathrm{d}x-\int_{y_{j-\frac{1}{2}}}^{y_{j-\frac{1}{2}}}uT\Big|_{w,c}\mathrm{d}y$$

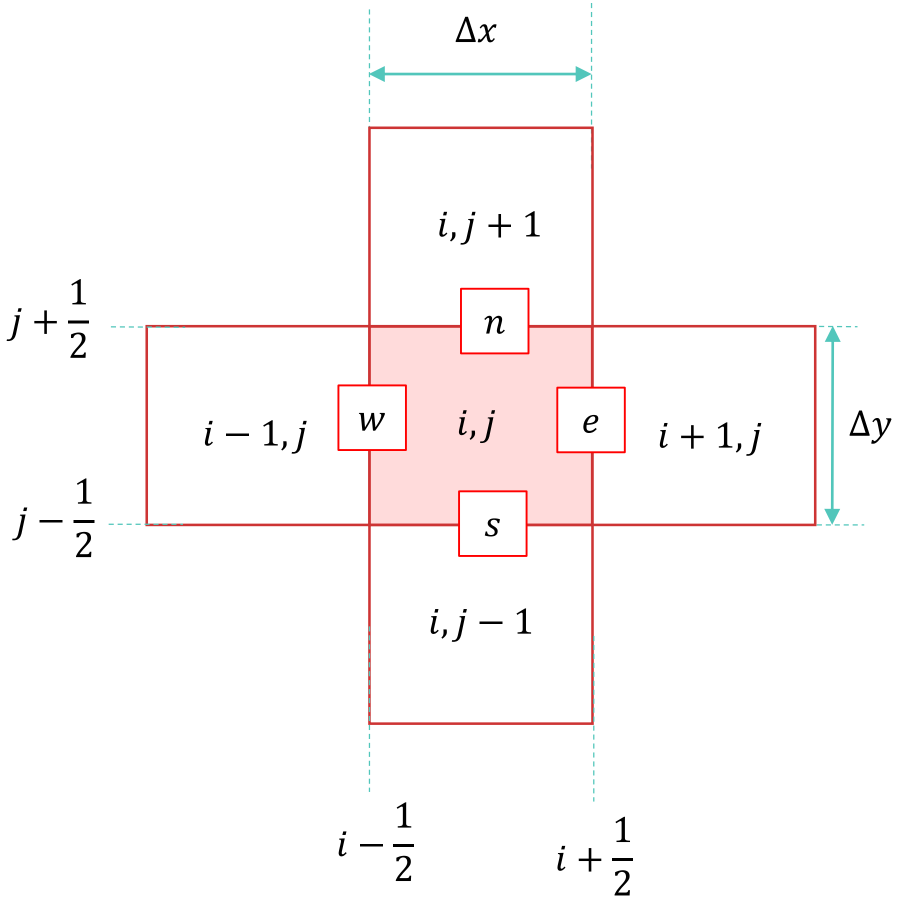
معادله انرژی

حجم محدود

$$\oint_{\partial A_{i,j}} \vec{V}T.\hat{n}\,\mathrm{d}l \approx -vT\Big|_{s,c}\Delta x + uT\Big|_{e,c}\Delta y + vT\Big|_{n,c}\Delta x - uT\Big|_{w,c}\Delta y$$

$$A_{i,j} = \Delta x\Delta y$$

$$\frac{1}{A_{i,j}}\oint_{\partial A_{i,j}} \begin{bmatrix} uT \\ vT \end{bmatrix}.\hat{n}\,\mathrm{d}l = \frac{u_eT_e - u_wT_w}{\Delta x} + \frac{v_nT_n - v_sT_s}{\Delta y}$$



$$\frac{1}{A_{i,j}}\oint_{\partial A_{i,j}} \begin{bmatrix} \dfrac{\partial T}{\partial x} \\ \dfrac{\partial T}{\partial y} \end{bmatrix}.\hat{n}\,\mathrm{d}l = \frac{\left(\dfrac{\partial T}{\partial x}\right)_e - \left(\dfrac{\partial T}{\partial x}\right)_w}{\Delta x} + \frac{\left(\dfrac{\partial T}{\partial y}\right)_n - \left(\dfrac{\partial T}{\partial y}\right)_s}{\Delta y}$$

$$\frac{\partial \bar{T}_{i,j}}{\partial t} + \frac{u_eT_e - u_wT_w}{\Delta x} + \frac{v_nT_n - v_sT_s}{\Delta y} - \frac{1}{\mathrm{Re.\,Pr}}\left(\frac{\left(\dfrac{\partial T}{\partial x}\right)_e - \left(\dfrac{\partial T}{\partial x}\right)_w}{\Delta x} + \frac{\left(\dfrac{\partial T}{\partial y}\right)_n - \left(\dfrac{\partial T}{\partial y}\right)_s}{\Delta y}\right) - S_{i,j} = 0$$

معادله انرژی

حجم محدود

بیان شارها بر حسب مقادیر میانگین حجمی

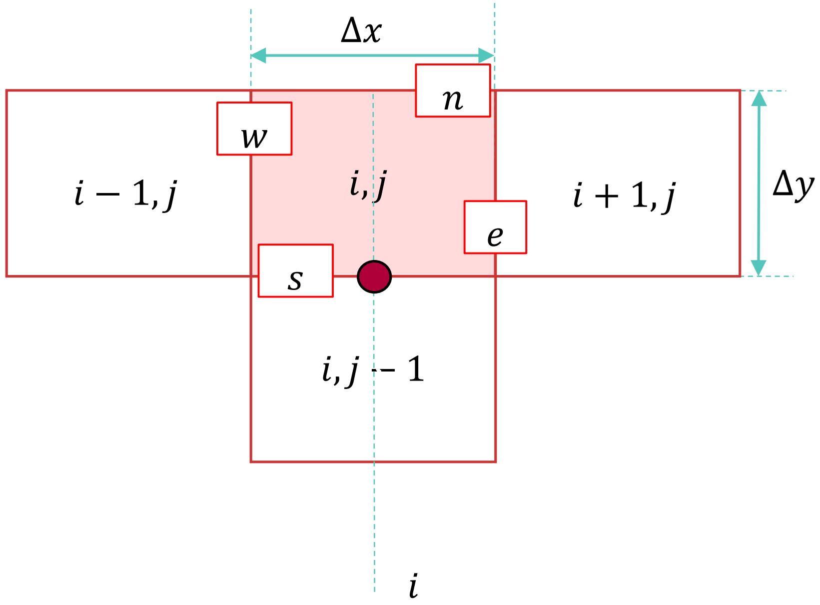
شارهای پخشی به صورت مرکزی

$$\left.\frac{\partial T}{\partial y}\right|_{s,c} = \frac{\bar{T}_{i,j} - \bar{T}_{i,j-1}}{\Delta y} + O(\Delta y^2)$$

$$\left.\frac{\partial T}{\partial x}\right|_{e,c} = \frac{\bar{T}_{i+1,j} - \bar{T}_{i,j}}{\Delta x} + O(\Delta x^2)$$

$$\left.\frac{\partial T}{\partial y}\right|_{n,c} = \frac{\bar{T}_{i,j+1} - \bar{T}_{i,j}}{\Delta y} + O(\Delta y^2)$$

$$\left.\frac{\partial T}{\partial x}\right|_{w,c} = \frac{\bar{T}_{i,j} - \bar{T}_{i-1,j}}{\Delta x} + O(\Delta x^2)$$



معادله انرژی

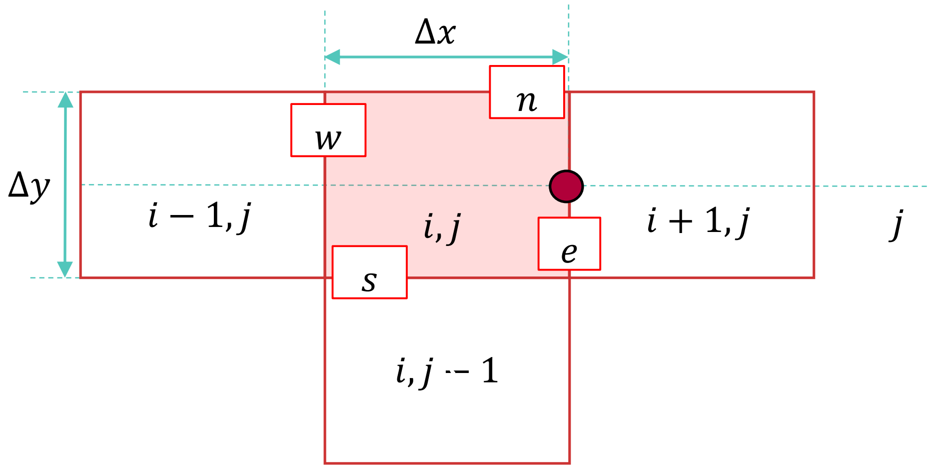
بیان شارها بر حسب مقادیر میانگین حجمی

شارهای جابجایی به صورت مرکزی یا upwind

بادسو مرتبه دو

$$T\Big|_{e,c} = \frac{3\bar{T}_{i,j} - \bar{T}_{i-1,j}}{\Delta x} + O(\Delta x^2)$$

در هر سلول باید به جهت سرعت یا $\vec{V}.\vec{n}$ دقت کرد.



در صورتی که پخش نسبت به جابجایی خیلی کوچک نباشد (عدد پکله).

مرکزی

$$T\Big|_{e,c} = \frac{\bar{T}_{i,j} + \bar{T}_{i+1,j}}{\Delta x} + O(\Delta x^2)$$

معادله انرژی

حجم محدود

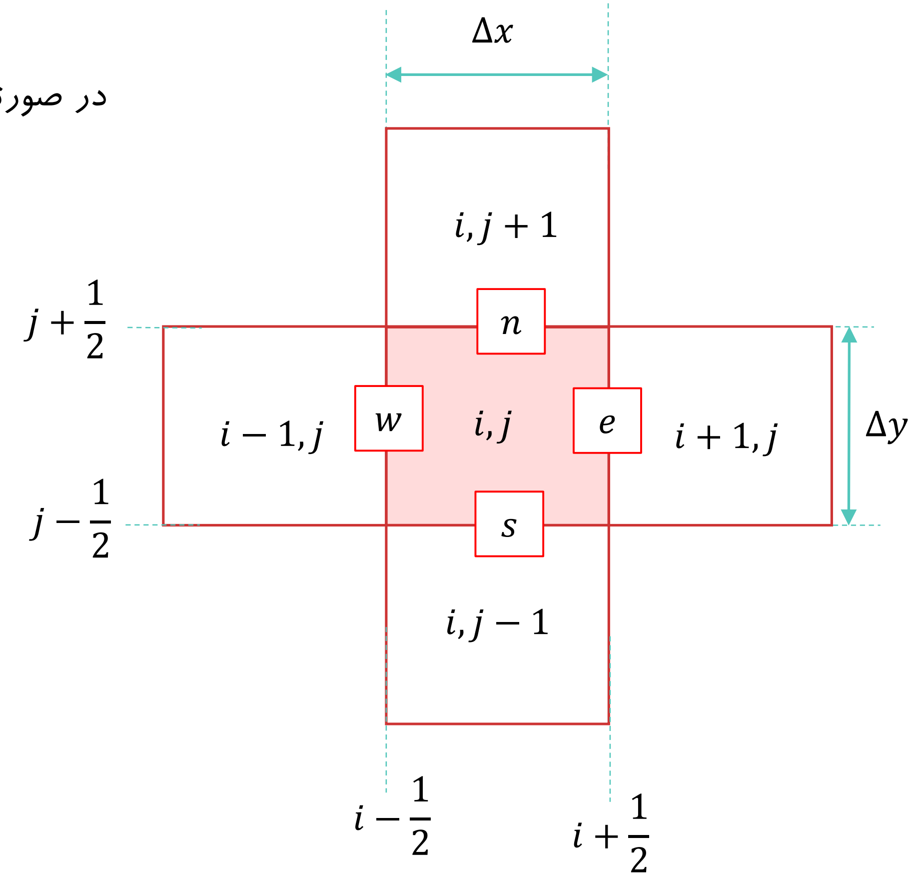
در صورتی که عدد رینولدز خیلی بزرگ نباشد.

$$T\Big|_{s,c} = \frac{\bar{T}_{i,j} + \bar{T}_{i,j-1}}{2} + O(\Delta y^2)$$

$$T\Big|_{e,c} = \frac{\bar{T}_{i+1,j} + \bar{T}_{i,j}}{2} + O(\Delta x^2)$$

$$T\Big|_{n,c} = \frac{\bar{T}_{i,j+1} + \bar{T}_{i,j}}{2} + O(\Delta y^2)$$

$$T\Big|_{w,c} = \frac{\bar{T}_{i,j} + \bar{T}_{i-1,j}}{2} + O(\Delta x^2)$$



$$\frac{d\bar{T}_{i,j}}{dt} + \frac{u_e(\bar{T}_{i,j} + \bar{T}_{i+1,j}) - u_w(\bar{T}_{i-1,j} + \bar{T}_{i,j})}{2\Delta x} + \frac{v_n(\bar{T}_{i,j} + \bar{T}_{i,j+1}) - v_s(\bar{T}_{i,j-1} + \bar{T}_{i,j})}{2\Delta y}$$

$$u_e = \frac{\bar{u}_{i,j} + \bar{u}_{i+1,j}}{2}$$

$$-\frac{1}{\mathrm{Re.\,Pr}}\left(\frac{\bar{T}_{i-1,j} - 2\bar{T}_{i,j} + \bar{T}_{i+1,j}}{\Delta x^2} + \frac{\bar{T}_{i,j-1} - 2\bar{T}_{i,j} + \bar{T}_{i,j+1}}{\Delta y^2}\right) - S_{i,j} = 0$$

صورت نیمه گسسته

$$\frac{\partial u}{\partial x}\Big|_c = \frac{\bar{u}_{i+1,j} - \bar{u}_{i-1,j}}{2\Delta x}$$

روش صریح اویلر

$$\frac{\mathrm{d}\bar{T}_{i,j}}{\mathrm{d}t} = -\frac{u_e\left(\bar{T}_{i,j}+\bar{T}_{i+1,j}\right)-u_w\left(\bar{T}_{i-1,j}+\bar{T}_{i,j}\right)}{2\Delta x} - \frac{v_n\left(\bar{T}_{i,j}+\bar{T}_{i,j+1}\right)-v_s\left(\bar{T}_{i,j-1}+\bar{T}_{i,j}\right)}{2\Delta y}$$

$$+\frac{1}{\mathrm{Re.\,Pr}}\left(\frac{\bar{T}_{i-1,j}-2\bar{T}_{i,j}+\bar{T}_{i+1,j}}{\Delta x^2}+\frac{\bar{T}_{i,j-1}-2\bar{T}_{i,j}+\bar{T}_{i,j+1}}{\Delta y^2}\right)+S_{i,j}$$

$$\frac{\bar{T}_{i,j}^{n+1}-\bar{T}_{i,j}^{n}}{\Delta t} = -\frac{u_e\left(\bar{T}_{i,j}^{n}+\bar{T}_{i+1,j}^{n}\right)-u_w\left(\bar{T}_{i-1,j}^{n}+\bar{T}_{i,j}^{n}\right)}{2\Delta x} - \frac{v_n\left(\bar{T}_{i,j}^{n}+\bar{T}_{i,j+1}^{n}\right)-v_s\left(\bar{T}_{i,j-1}^{n}+\bar{T}_{i,j}^{n}\right)}{2\Delta y}$$

$$+\frac{1}{\mathrm{Re.\,Pr}}\left(\frac{\bar{T}_{i-1,j}^{n}-2\bar{T}_{i,j}^{n}+\bar{T}_{i+1,j}^{n}}{\Delta x^2}+\frac{\bar{T}_{i,j-1}^{n}-2\bar{T}_{i,j}^{n}+\bar{T}_{i,j+1}^{n}}{\Delta y^2}\right)+S_{i,j}$$

$$\frac{\bar{T}_{i,j}^{n+1}-\bar{T}_{i,j}^{n}}{\Delta t} = R_{i,j}^{n} \qquad \bar{T}_{i,j}^{n+1} = R_{i,j}^{n}\Delta t+\bar{T}_{i,j}^{n}+O(\Delta t)$$

روش ضمنی اویلر

$$\frac{\mathrm{d}\bar{T}_{i,j}}{\mathrm{d}t}=\frac{\bar{T}_{i,j}^{n}-\bar{T}_{i,j}^{n-1}}{\Delta t}$$

$$\frac{\bar{T}_{i,j}^{n}-\bar{T}_{i,j}^{n-1}}{\Delta t}=R_{i,j}^{n} \qquad \bar{T}_{i,j}^{n}=\bar{T}_{i,j}^{n-1}+R_{i,j}^{n}\Delta t$$

$$\bar{T}_{i,j}^{n}=\bar{T}_{i,j}^{n-1}+\delta\bar{T}_{i,j}^{n} \qquad \delta\bar{T}_{i,j}^{n}=R_{i,j}^{n}\Delta t$$

$$\frac{\bar{T}_{i,j}^{n}-\bar{T}_{i,j}^{n-1}}{\Delta t}=-\frac{u_e\left(\bar{T}_{i,j}^{n}+\bar{T}_{i+1,j}^{n}\right)-u_w\left(\bar{T}_{i-1,j}^{n}+\bar{T}_{i,j}^{n}\right)}{2\Delta x}-\frac{v_n\left(\bar{T}_{i,j}^{n}+\bar{T}_{i,j+1}^{n}\right)-v_s\left(\bar{T}_{i,j-1}^{n}+\bar{T}_{i,j}^{n}\right)}{2\Delta y}$$

$$+\frac{1}{\mathrm{Re.\,Pr}}\left(\frac{\bar{T}_{i-1,j}^{n}-2\bar{T}_{i,j}^{n}+\bar{T}_{i+1,j}^{n}}{\Delta x^2}+\frac{\bar{T}_{i,j-1}^{n}-2\bar{T}_{i,j}^{n}+\bar{T}_{i,j+1}^{n}}{\Delta y^2}\right)+S_{i,j}$$

روش ضمنی اویلر

$$\frac{\delta \bar{T}^{n}_{i,j}}{\Delta t} = -\frac{u_e\left(\left(\bar{T}^{n-1}_{i,j}+\delta \bar{T}^{n}_{i,j}\right)+\left(\bar{T}^{n-1}_{i+1,j}+\delta \bar{T}^{n}_{i+1,j}\right)\right)-u_w\left(\left(\bar{T}^{n-1}_{i-1,j}+\delta \bar{T}^{n}_{i-1,j}\right)+\left(\bar{T}^{n-1}_{i,j}+\delta \bar{T}^{n}_{i,j}\right)\right)}{2\Delta x}$$

$$-\frac{v_n\left(\left(\bar{T}^{n-1}_{i,j}+\delta \bar{T}^{n}_{i,j}\right)+\left(\bar{T}^{n-1}_{i,j+1}+\delta \bar{T}^{n}_{i,j+1}\right)\right)-v_s\left(\left(\bar{T}^{n-1}_{i,j-1}+\delta \bar{T}^{n}_{i,j-1}\right)+\left(\bar{T}^{n-1}_{i,j}+\delta \bar{T}^{n}_{i,j}\right)\right)}{2\Delta y}$$

$$+\frac{1}{\mathrm{Re.\,Pr}}\left(\frac{\left(\bar{T}^{n-1}_{i-1,j}+\delta \bar{T}^{n}_{i-1,j}\right)-2\left(\bar{T}^{n-1}_{i,j}+\delta \bar{T}^{n}_{i,j}\right)+\left(\bar{T}^{n-1}_{i+1,j}+\delta \bar{T}^{n}_{i+1,j}\right)}{\Delta x^2}\right.$$

$$\left.+\frac{\left(\bar{T}^{n-1}_{i,j-1}+\delta \bar{T}^{n}_{i,j-1}\right)-2\left(\bar{T}^{n-1}_{i,j}+\delta \bar{T}^{n}_{i,j}\right)+\left(\bar{T}^{n-1}_{i,j+1}+\delta \bar{T}^{n}_{i,j+1}\right)}{\Delta y^2}\right)+S_{i,j}$$

$$\delta T^{n}_{ij}\left(1+\frac{u_e\Delta t}{2\Delta x}-\frac{u_w\Delta t}{2\Delta x}+\frac{2\Delta t}{\mathrm{Re.\,Pr.}\,\Delta x^2}+\frac{v_n\Delta t}{2\Delta y}-\frac{v_s\Delta t}{2\Delta y}+\frac{2\Delta t}{\mathrm{Re.\,Pr.}\,\Delta y^2}\right)$$

$$+\delta T^{n}_{i+1,j}\left(\frac{u_e\Delta t}{2\Delta x}-\frac{\Delta t}{\mathrm{Re.\,Pr.}\,\Delta x^2}\right)+\delta T^{n}_{i-1.j}\left(-\frac{u_w\Delta t}{2\Delta x}-\frac{\Delta t}{\mathrm{Re.\,Pr.}\,\Delta x^2}\right)$$

$$+\delta T^{n}_{i,j+1}\left(\frac{v_n\Delta t}{2\Delta y}-\frac{\Delta t}{\mathrm{Re.\,Pr.}\,\Delta y^2}\right)+\delta T^{n}_{i,j-1}\left(-\frac{v_s\Delta t}{2\Delta y}-\frac{\Delta t}{\mathrm{Re.\,Pr.}\,\Delta y^2}\right)=R^{n-1}_{i,j}\times\Delta t$$

معادله انرژی

روش ضمنی اویلر

$$\delta T^{n}_{ij}\left(1 + a_x\Delta t + a_y\Delta t\right) + \delta T^{n}_{i+1,j}(b_x\Delta t) + \delta T^{n}_{i-1.j}(c_x\Delta t) + \delta T^{n}_{i,j+1}\left(b_y\Delta t\right) + \delta T^{n}_{i,j-1}\left(c_y\Delta t\right) = R^{n-1}_{i,j} \times \Delta t$$

$$a_x = \frac{u_e}{2\Delta x} - \frac{u_w}{2\Delta x} + \frac{2}{\mathrm{Re.Pr.}\Delta x^2}$$

$$a_y = \frac{v_n}{2\Delta y} - \frac{v_s}{2\Delta y} + \frac{2}{\mathrm{Re.Pr.}\Delta y^2}$$

$$b_x = \frac{u_e}{2\Delta x} - \frac{1}{\mathrm{Re.Pr.}\Delta x^2}$$

$$b_y = \frac{v_n}{2\Delta y} - \frac{1}{\mathrm{Re.Pr.}\Delta y^2}$$

$$c_x = -\frac{u_w}{2\Delta x} - \frac{}{\mathrm{Re.Pr.}\Delta x^2}$$

$$c_y = -\frac{v_s}{2\Delta y} - \frac{1}{\mathrm{Re.Pr.}\Delta y^2}$$

معادله انرژی

روش ضمنی اویلر

$$\delta T^{n}_{i,j-1}\left(c_y\Delta t\right)+\delta T^{n}_{i-1.j}\left(c_x\Delta t\right)+\delta T^{n}_{ij}\left(1+a_x\Delta t+a_y\Delta t\right)+\delta T^{n}_{i+1,j}\left(b_x\Delta t\right)+\delta T^{n}_{i,j+1}\left(b_y\Delta t\right)=R^{n-1}_{i,j}\times\Delta t$$

$$\begin{bmatrix}
1+a_x\Delta t+a_y\Delta t & b_x\Delta t & 0 & \cdots & 0 & b_y\Delta t & 0 & 0 & 0\\
c_x & 1+a_x\Delta t+a_y\Delta t & b_{x\Delta t} & & & & \ddots & 0 & 0\\
0 & c_x & & & & & & b_y\Delta t & 0\\
\vdots & & & & & & & & b_y\Delta t\\
0 & & & \ddots & \ddots & \ddots & & & \\
c_{y\Delta t} & & & & & & & & \\
0 & \ddots & & & & & & & \\
0 & 0 & c_y\Delta t & & & & & & b_x\\
0 & 0 & 0 & c_y\Delta t & & & & c_x\Delta t & 1+a_x\Delta t+a_y\Delta t
\end{bmatrix}
\begin{bmatrix}
\delta T^{n}_{1,1}\\ \delta T^{n}_{2,1}\\ \delta T^{n}_{3,1}\\ \vdots\\ \delta T^{n}_{N-1,M}\\ \delta T^{n}_{N,M}
\end{bmatrix}
=
\begin{bmatrix}
R^{n-1}_{1,1}\times\Delta t\\ R^{n-1}_{2,1}\times\Delta t\\ R^{n-1}_{3,1}\times\Delta t\\ \vdots\\ R^{n-1}_{N-1,M}\times\Delta t\\ R^{n-1}_{N,M}\times\Delta t
\end{bmatrix}$$

$$A_{P\times P}\times\overrightarrow{\delta T}^{n}{}_{P\times 1}=\vec{R}^{n-1}{}_{P\times 1}\times\Delta t \qquad P=N\times M$$

معادله انرژی

روش ضمنی اویلر

روش فاکتورگیری تقریبی (Approximate Factorization)

$$A \times \overrightarrow{\delta T}^{n} = \vec{R}^{n-1} \times \Delta t$$

$$A = I_{P\times P} + B_x \times \Delta t + B_y \times \Delta t$$

$$B_x = \text{tridaig}(a_x, b_x, c_x)$$

$$B_y = \text{tridaig}(a_y, b_y, c_y)$$

$$B_x = \begin{bmatrix} a_x & b_x & & & & \\ c_x & a_x & b_x & & & \\ & c_x & a_x & b_x & & \\ & & \ddots & \ddots & \ddots & \\ & & & c_x & a_x & b_x \\ & & & & c_x & a_x \end{bmatrix}_{P\times P}$$

$$B_y = \begin{bmatrix} a_y & & & b_y & & \\ & a_y & & & \ddots & \\ & & a_y & & & b_y \\ c_y & & \ddots & \ddots & \ddots & \\ & \ddots & & & a_y & \\ & & c_y & & & a_y \end{bmatrix}_{P\times P}$$

$$\left(I + B_x\Delta t + B_y\Delta t\right) \times \overrightarrow{\delta T}^{n} = \vec{R}^{n-1} \times \Delta t$$

معادله انرژی

روش ضمنی اویلر

روش فاکتورگیری تقریبی (Approximate Factorization)

$$I + B_x\Delta t + B_y\Delta t = I + B_x\Delta t + B_y\Delta t + B_x \times B_y \times \Delta t^2 - B_x \times B_y \times \Delta t^2$$

$$I + B_x\Delta t + B_y\Delta t = I + B_x\Delta t + B_y\Delta t + B_x \times B_y \times \Delta t^2 + O(\Delta t^2) \cong I + B_x\Delta t + B_y\Delta t + B_x \times B_y \times \Delta t^2$$

$$I + B_x\Delta t + B_y\Delta t \cong (I + B_x\Delta t) \times (I + B_y\Delta t)$$

$$(I + B_x\Delta t) \times (I + B_y\Delta t) \times \overrightarrow{\delta T}^n = \vec{R}^{n-1} \times \Delta t$$

$$\overrightarrow{\delta T}^{n-\frac{1}{2}} = (I + B_y\Delta t) \times \overrightarrow{\delta T}^n$$

$$(I + B_x\Delta t) \times \overrightarrow{\delta T}^{n-\frac{1}{2}} = \vec{R}^{n-1} \times \Delta t$$

معادله انرژی

روش ضمنی اویلر

روش فاکتورگیری تقریبی (Approximate Factorization)

روند حل

$$(I + T_x \Delta t) \times \overrightarrow{\delta T}^{n-\frac{1}{2}} = \vec{R}^{n-1} \times \Delta t$$

$$\left(I + T_y \Delta t\right) \times \overrightarrow{\delta T}^{n} = \overrightarrow{\delta T}^{n-\frac{1}{2}}$$

$$\vec{T}^{n} = \vec{T}^{n-1} + \overrightarrow{\delta T}^{n}$$

چالش: $I + T_y \Delta t$ با الگوریتم توماس قابل حل نیست.

راهکار: ۱. استفاده از روش جهت متناوب در کل دامنه حل

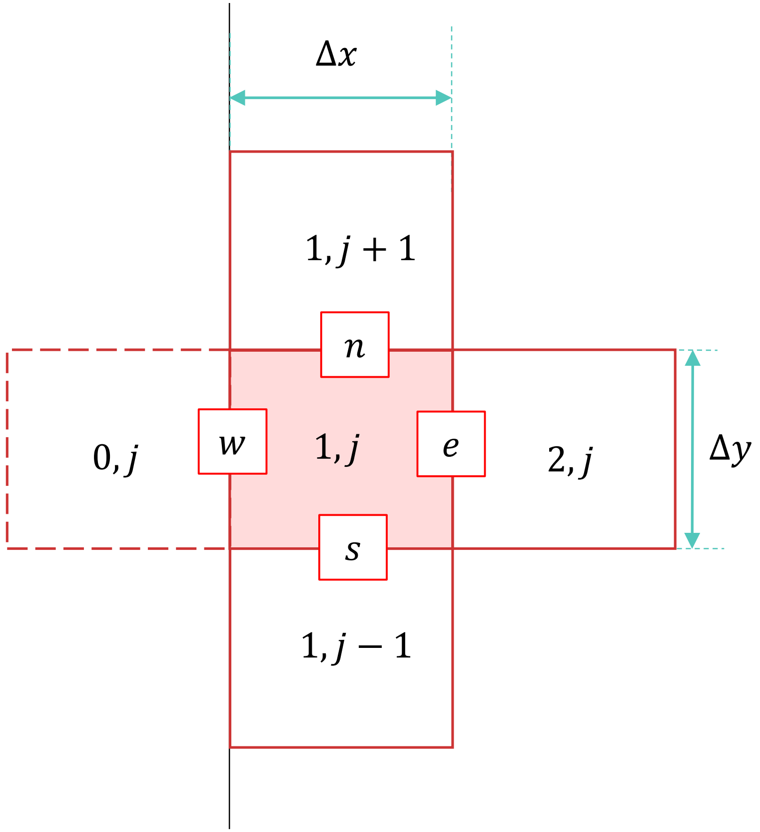
۲. ابتدا یک گام ردیف به ردیف حل شود سپس یک گام ستون به ستون (سهولت در اعمال شرط مرزی)

معادله انرژی

روش ضمنی اویلر

شرایط مرزی

سلول مجازی



$$T(0, y, t) = T_1(y, t)$$ شرط دیریشله

$$T_w\Big|_{1,j}^{n} = T_1(y_j, t_n)$$

$$\frac{\bar{T}_{0,j}^{n} + \bar{T}_{1,j}^{n}}{2} = T_1(y_j, t_n) \qquad \frac{\bar{T}_{0,j}^{n-1} + \bar{T}_{1,j}^{n-1}}{2} = T_1(y_j, t_{n-1})$$

$$\frac{\left(\bar{T}_{0,j}^{n} - \bar{T}_{0,j}^{n-1}\right) + \left(\bar{T}_{1,j}^{n} - \bar{T}_{1,j}^{n-1}\right)}{2} = T_1\left(y_j, t_n\right) - T_1\left(y_j, t_{n-1}\right)$$

$$\frac{\delta T_{0,j}^{n} + \delta T_{1,j}^{n}}{2} = T_1\left(y_j, t_n\right) - T_1\left(y_j, t_{n-1}\right)$$

$$\delta T_{0,j}^{n} = -\delta T_{1,j}^{n} + 2\left(T_1\left(y_j, t_n\right) - T_1\left(y_j, t_{n-1}\right)\right)$$

$$\delta T_{0,j}^{n} = -\delta T_{1,j}^{n}$$ حل پایا

روش ضمنی اویلر

شرایط مرزی

سلول مجازی

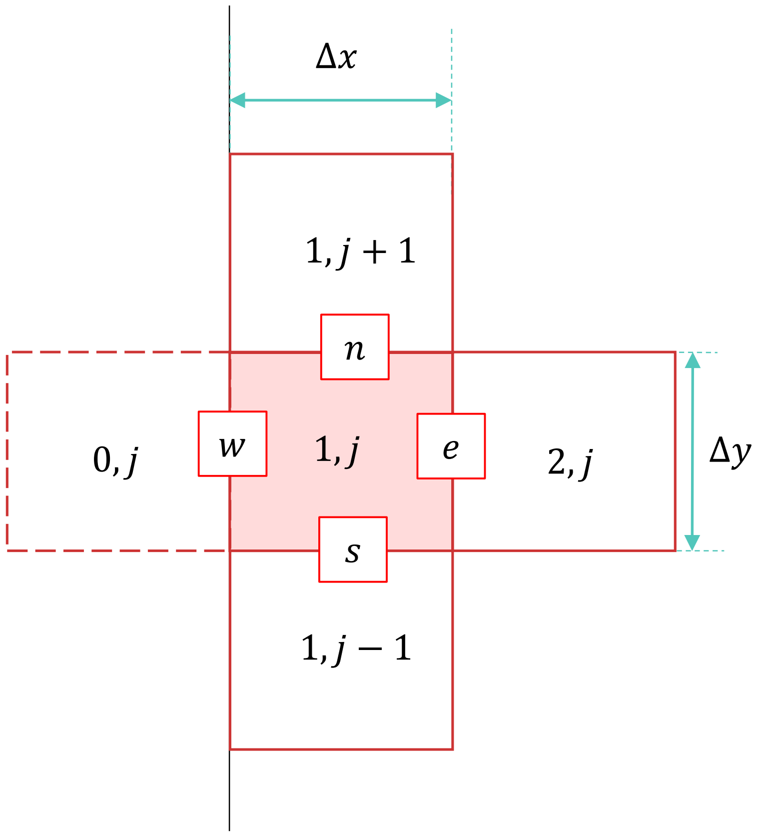
$$\frac{\partial T}{\partial x}(0,y,t) = T_1(y,t)$$ شرط نیومن

$$\frac{\bar{T}^{n}_{1,j} - \bar{T}^{n}_{0,j}}{\Delta x} = T_1(y_j,t_n) \qquad \frac{\bar{T}^{n-1}_{1,j} - \bar{T}^{n-1}_{0,j}}{\Delta x} = T_1(y_j,t_{n-1})$$

$$\frac{\left(\bar{T}^{n}_{0,j} - \bar{T}^{n-1}_{0,j}\right) - \left(\bar{T}^{n}_{1,j} - \bar{T}^{n-1}_{1,j}\right)}{\Delta x} = T_1\left(y_j,t_{n-1}\right) - T_1(y_j,t_n)$$

$$\frac{\delta T^{n}_{0,j} - \delta T^{n}_{1,j}}{\Delta x} = T_1\left(y_j,t_{n-1}\right) - T_1(y_j,t_n)$$

$$\delta T^{n}_{0,j} = \delta T^{n}_{1,j} + \Delta x\left(T_1\left(y_j,t_{n-1}\right) - T_1(y_j,t_n)\right)$$

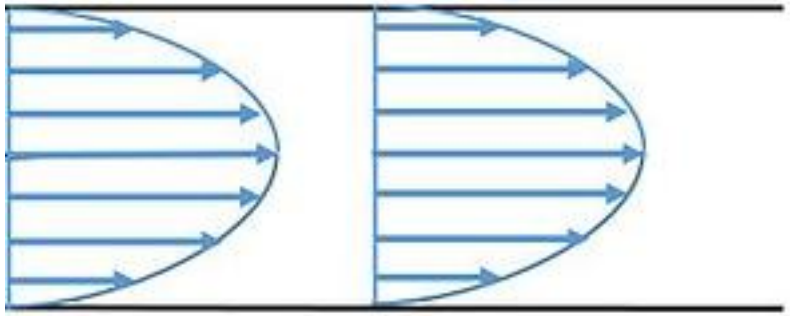


$$\delta T^{n}_{0,j} = \delta T^{n}_{1,j}$$ حل پایا

مثال

$$T = 1$$

$$T = T_i(y)$$



$$\frac{\partial T}{\partial x} = 0$$

$$T = 0$$

$$T_i(y) = y + 3\mathrm{Pr.\,Ec.}\,U^2(1 - (1 - 2\mathrm{y})^4)$$

$$u(x, y) = 12Uy(1 - y)$$

$$v(x, y) = 0$$

معادله $\psi - \omega$

جریان دو بعدی سیال تراکم‌ناپذیر

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\frac{\partial u^*}{\partial t^*} + u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}}\right)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

$$\frac{\partial v^*}{\partial t^*} + u^*\frac{\partial v^*}{\partial x^*} + v^*\frac{\partial v^*}{\partial y^*} = -\frac{\partial p^*}{\partial y^*} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}}\right)$$

در سیال تراکم‌ناپذیر معادله پیوستگی مستقل از فشار است.

معادله $\psi - \omega$

حذف فشار از معادلات

$$\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)\right)$$

$$\frac{\partial^2 u}{\partial y\partial t}+\frac{\partial u}{\partial y}\frac{\partial u}{\partial x}+u\frac{\partial^2 u}{\partial y\partial x}+\frac{\partial v}{\partial y}\frac{\partial u}{\partial y}+v\frac{\partial^2 u}{\partial y^2}=-\frac{\partial^2 p}{\partial y\partial x}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^3 u}{\partial y\partial x^2}+\frac{\partial^3 u}{\partial y^3}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)\right)$$

$$\frac{\partial^2 v}{\partial x\partial t}+\frac{\partial u}{\partial x}\frac{\partial v}{\partial x}+u\frac{\partial^2 v}{\partial x^2}+\frac{\partial v}{\partial x}\frac{\partial v}{\partial y}+v\frac{\partial^2 v}{\partial x\partial y}=-\frac{\partial^2 p}{\partial x\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^3 v}{\partial x^3}+\frac{\partial^3 v}{\partial x\partial y^2}\right)$$

معادله $\psi - \omega$

$$\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+\frac{\partial u}{\partial x}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+u\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+\frac{\partial v}{\partial y}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+v\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)=$$

$$\frac{1}{\mathrm{Re}}\left(\frac{\partial^2}{\partial x^2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+\frac{\partial^2}{\partial y^2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)\right)$$

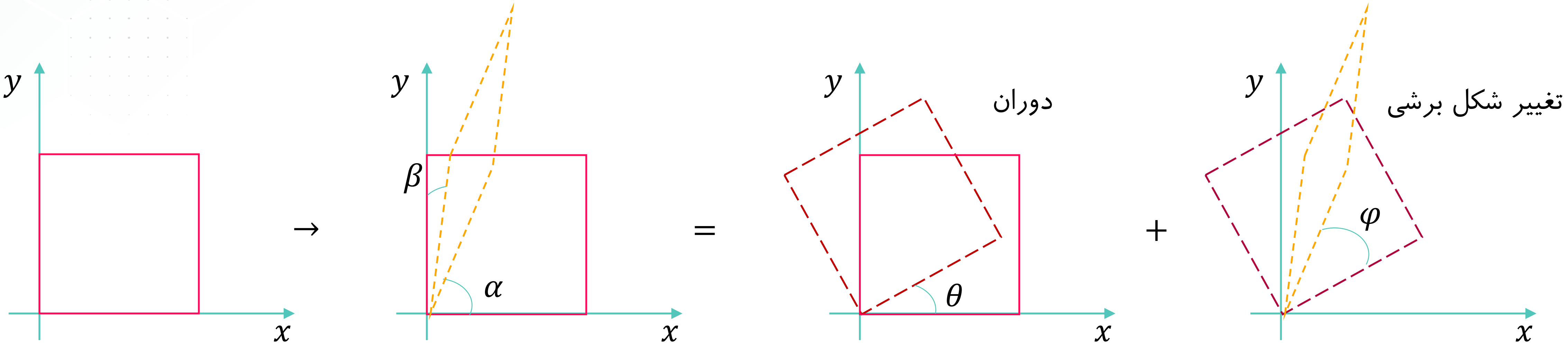
$$\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+u\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+v\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)=$$

$$\frac{1}{\mathrm{Re}}\left(\frac{\partial^2}{\partial x^2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+\frac{\partial^2}{\partial y^2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)\right)$$

معادله $\psi - \omega$

تاوایی (چرخش)

$$\vec{\omega} = \nabla \times \vec{V} = \begin{vmatrix} i & j & k \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = \begin{vmatrix} i & j & k \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & 0 \\ u & v & 0 \end{vmatrix} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) k$$



$$\theta = \frac{\alpha - \beta}{2}$$

$$\varphi = \frac{\alpha + \beta}{2}$$

$$\dot{\theta} = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

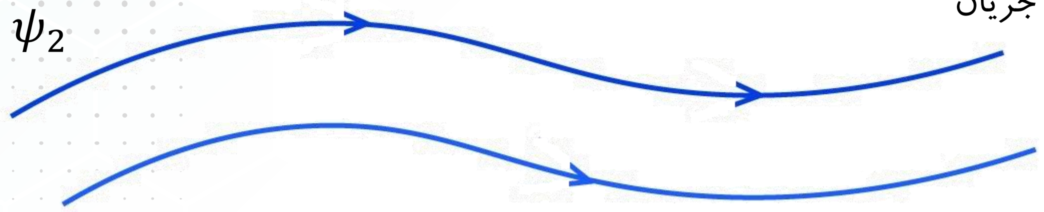
$$\omega = 2\dot{\theta}$$

معادله $\psi - \omega$

تابع جریان

خطوط جریان

$\psi = \psi_2$



$\psi = \psi_1$

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy = 0 \qquad \frac{dy}{dx} = \frac{-\frac{\partial \psi}{\partial x}}{\frac{\partial \psi}{\partial y}} = \frac{v}{u}$$

$$u = \frac{\partial \psi}{\partial y}$$

$$Q_p = \frac{Q_v}{\text{width}} = \Delta\psi$$

$$v = -\frac{\partial \psi}{\partial x}$$

معادله $\psi - \omega$

معادله مستقل از فشار

همواره برقرار

$$\nabla \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial x}\right) = 0$$

$$\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + u\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + v\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{\mathrm{Re}}\left(\frac{\partial^2}{\partial x^2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + \frac{\partial^2}{\partial y^2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\right)$$

$$\frac{\partial \omega}{\partial t} + u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right)$$

$$\boxed{\frac{\partial \omega}{\partial t} + \frac{\partial \psi}{\partial y}\frac{\partial \omega}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \omega}{\partial y} = \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right)}$$

معادله اول (تفاضل محدود)

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x}$$

معادله $\psi - \omega$

معادله مستقل از فشار

$$\frac{\partial \omega}{\partial t}+\frac{\partial (u\omega)}{\partial x}+\frac{\partial (v\omega)}{\partial y}=\frac{\partial}{\partial x}\left(\frac{1}{\mathrm{Re}}\frac{\partial \omega}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{\mathrm{Re}}\frac{\partial \omega}{\partial y}\right)$$

معادله اول (حجم محدود)

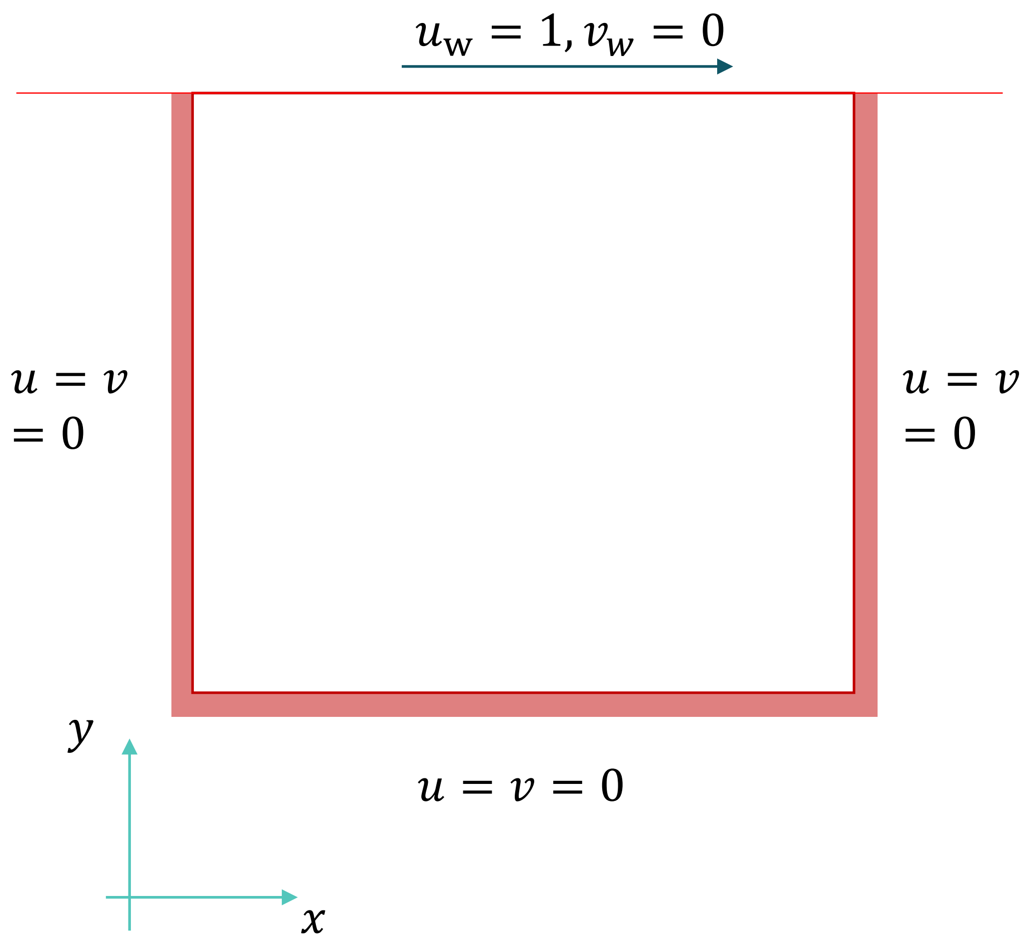
$$\boxed{\frac{\partial \omega}{\partial t}+\frac{\partial}{\partial x}\left(\omega\frac{\partial \psi}{\partial y}\right)+\frac{\partial}{\partial y}\left(-\omega\frac{\partial \psi}{\partial x}\right)=\frac{\partial}{\partial x}\left(\frac{1}{\mathrm{Re}}\frac{\partial \omega}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{\mathrm{Re}}\frac{\partial \omega}{\partial y}\right)}$$

$$\omega=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}=\frac{\partial}{\partial x}\left(-\frac{\partial \psi}{\partial x}\right)-\frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial y}\right)$$

معادله دوم

$$\boxed{\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^2 \psi}{\partial y^2}=-\omega}$$

مسئله حفره درپوش متحرک (Lid-driven cavity)



شرط مرزی ψ

دیواره غربی و شرقی

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy$$

$$\frac{\partial \psi}{\partial y} = u$$

$$\frac{\partial \psi}{\partial x} = -v$$

$$d\psi = -\cancel{v}\,\cancel{d}x + \cancel{u}\,dy = 0$$

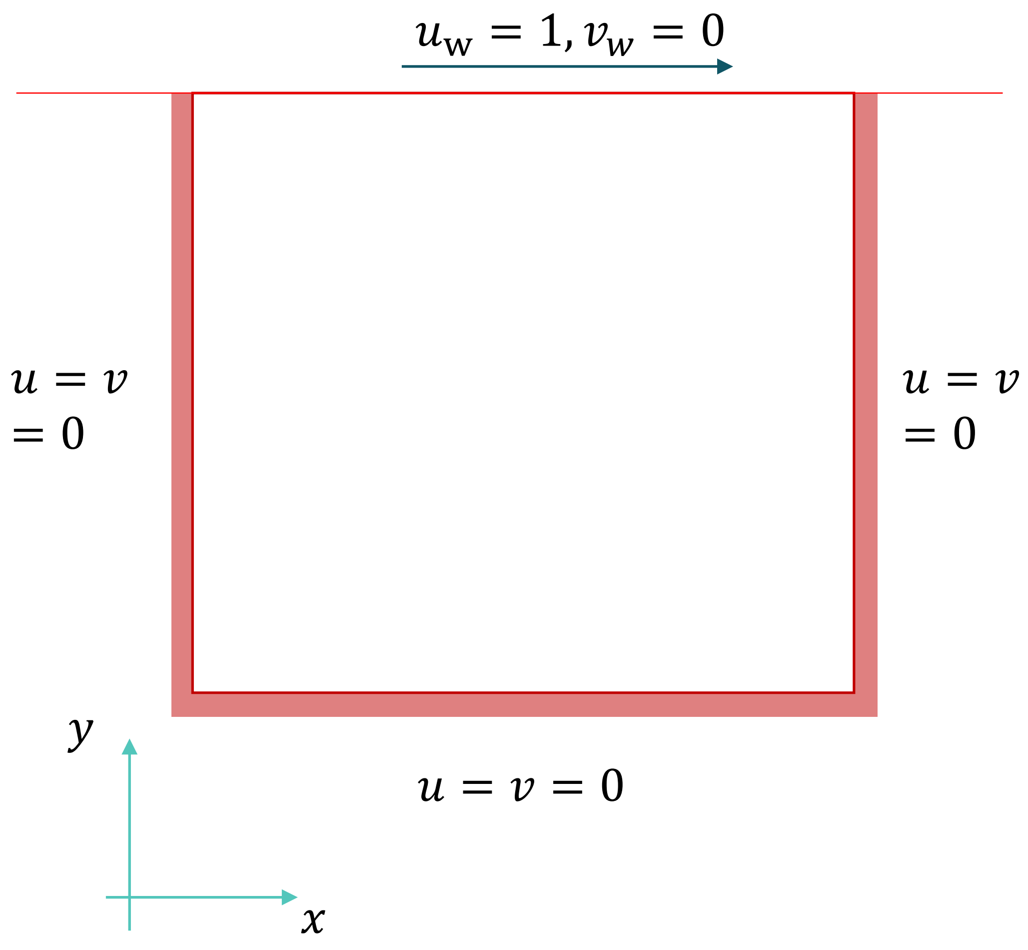
$$0 \quad 0 \quad\quad 0$$

$$\psi\big|_E = \psi_E(\text{const.})$$

$$\psi\big|_W = \psi_W(\text{const.})$$

شرایط مرزی

مسئله حفره درپوش متحرک (Lid-driven cavity)



شرط مرزی ψ

دیواره شمالی و جنوبی

$$d\psi=\frac{\partial\psi}{\partial x}dx+\frac{\partial\psi}{\partial y}dy$$

$$\frac{\partial\psi}{\partial y}=u$$

$$\frac{\partial\psi}{\partial x}=-v$$

$$d\psi=-\underbrace{\cancel{v}}_{0}dx+u\underbrace{\cancel{dy}}_{0}=0$$

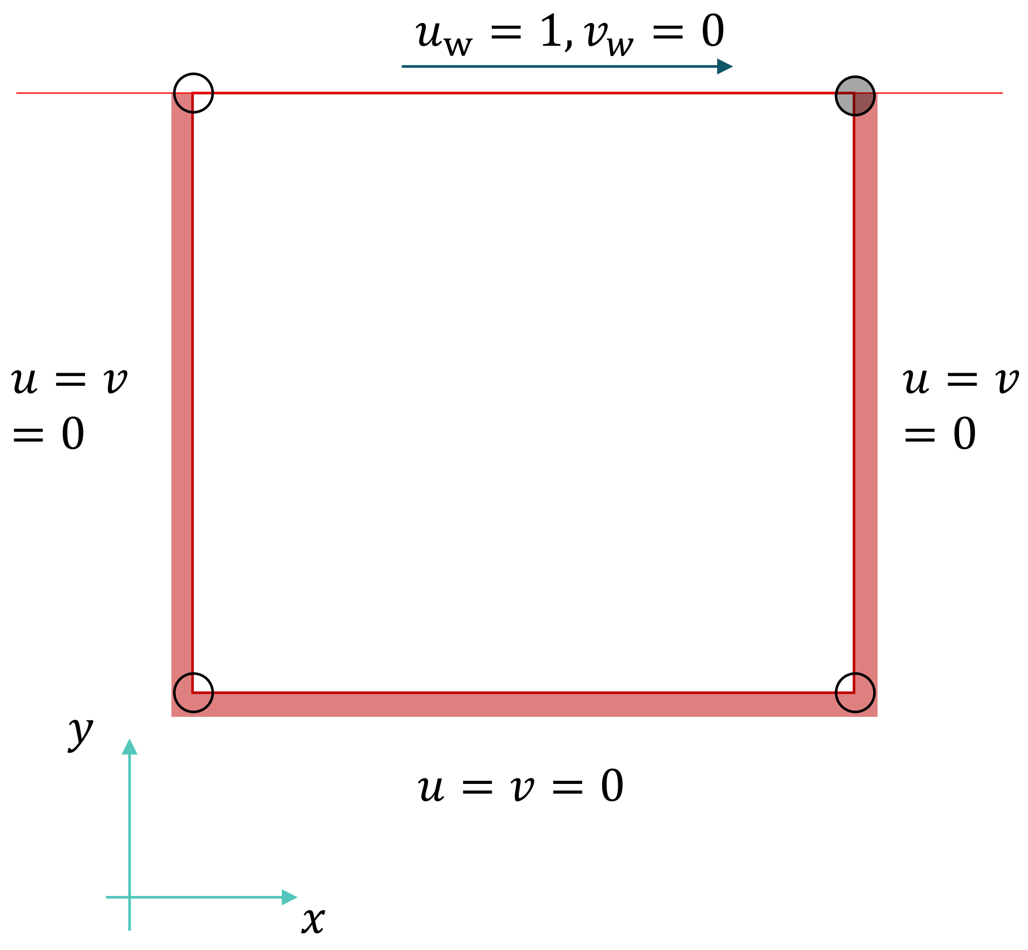
$$\psi\Big|_N=\psi_N(\text{const.})$$

$$\psi\Big|_S=\psi_S(\text{const.})$$

معادله $\psi - \omega$

شرایط مرزی

مسئله حفره درپوش متحرک (Lid-driven cavity)



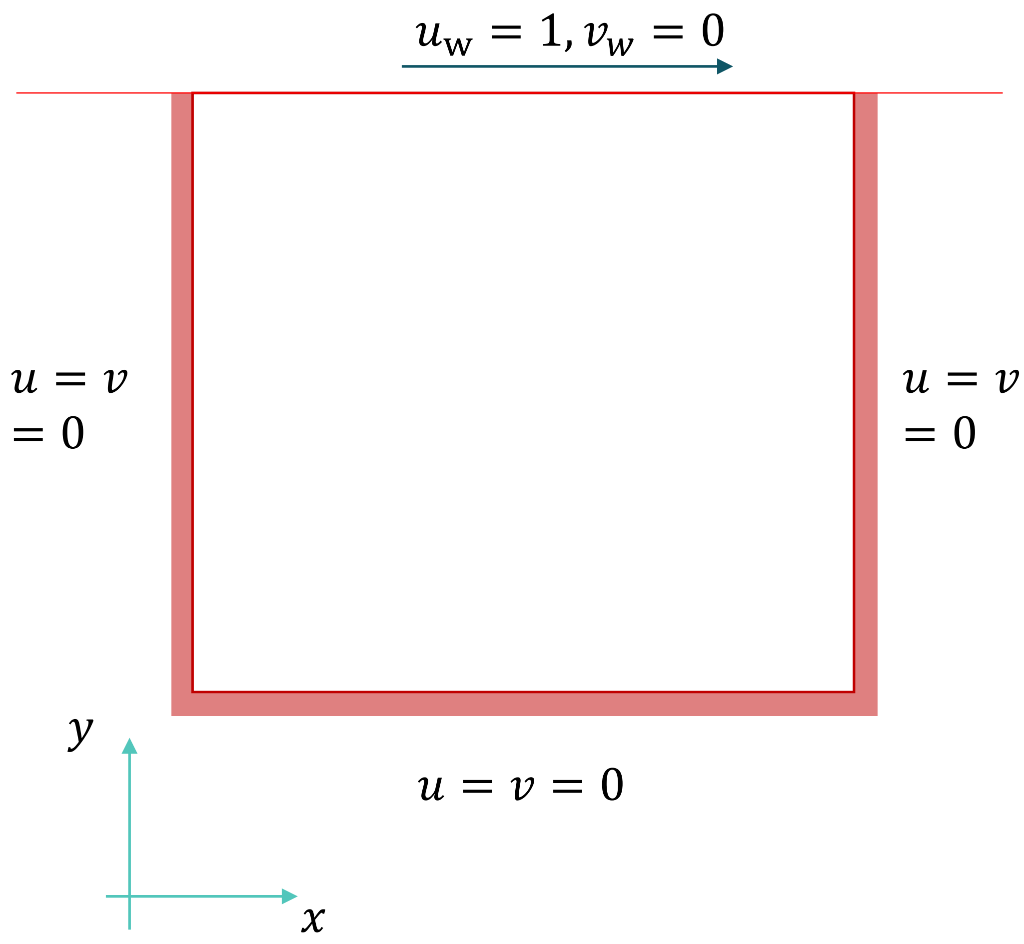
شرط مرزی ψ

$$\left.\begin{array}{l} \psi\big|_{NE} = \psi_N \\ \psi\big|_{NE} = \psi_E \end{array}\right\} \psi_N = \psi_E$$

$$\left.\begin{array}{l} \psi_N = \psi_E \\ \psi_E = \psi_S \\ \psi_S = \psi_W \\ \psi_W = \psi_N \end{array}\right\} \psi_N = \psi_E = \psi_S = \psi_W = \psi_{BC} = 0$$

شرایط مرزی

مسئله حفره درپوش متحرک (Lid-driven cavity)



شرط مرزی ω

$$\omega=-\frac{\partial^2\psi}{\partial x^2}-\frac{\partial^2\psi}{\partial y^2}$$

دیواره غربی و شرقی

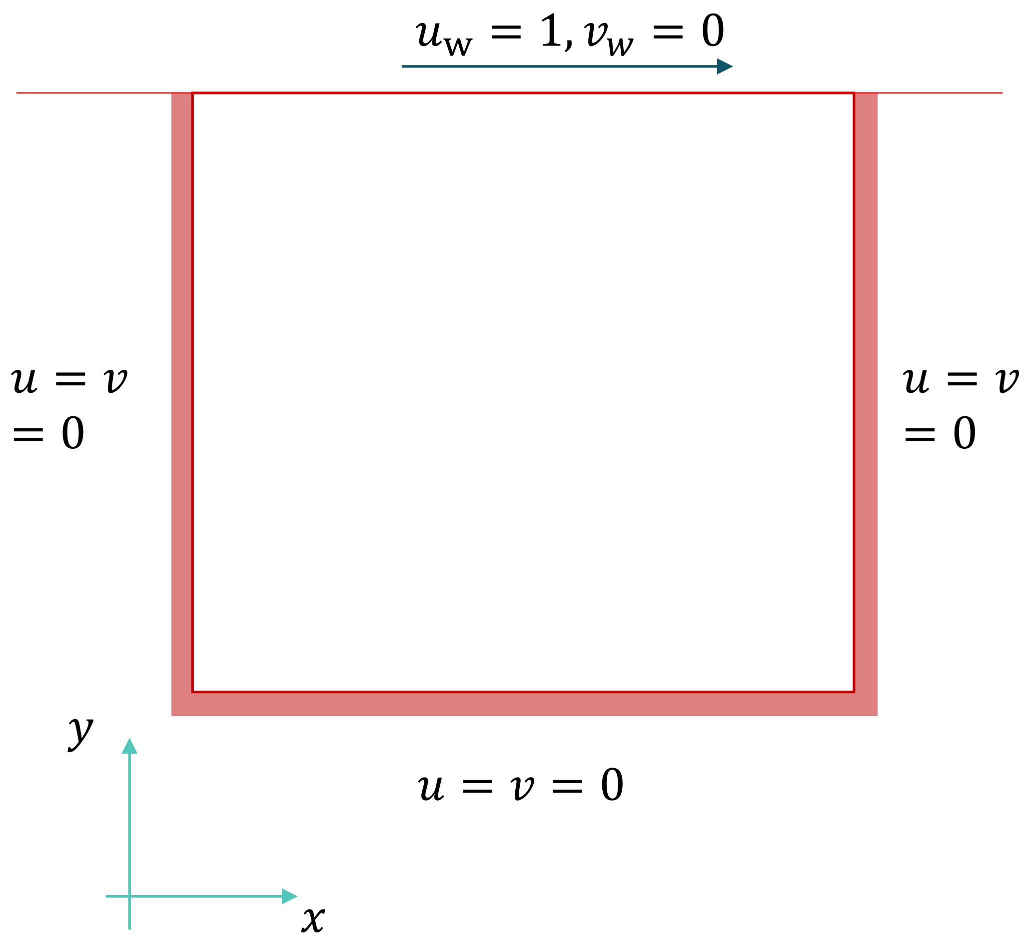
$$u=0 \qquad \frac{\partial u}{\partial y}=0$$

$$\frac{\partial^2\psi}{\partial y^2}=\frac{\partial}{\partial y}\left(\frac{\partial\psi}{\partial y}\right)=\frac{\partial u}{\partial y}=0$$

$$v=0 \qquad \frac{\partial v}{\partial x}\neq 0$$

$$\boxed{\omega=-\frac{\partial^2\psi}{\partial x^2}}$$

مسئله حفره درپوش متحرک (Lid-driven cavity)



شرط مرزی ω

$$\omega = -\frac{\partial^2\psi}{\partial x^2} - \frac{\partial^2\psi}{\partial y^2}$$

دیواره شمالی و جنوبی

$$v = 0 \qquad \frac{\partial v}{\partial x} = 0 \qquad \frac{\partial^2\psi}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial\psi}{\partial x}\right) = -\frac{\partial v}{\partial x} = 0$$

$$\frac{\partial u}{\partial y} \neq 0$$

$$\boxed{\omega = -\frac{\partial^2\psi}{\partial y^2}}$$

معادله $\psi - \omega$

شرایط مرزی

مسئله حفره درپوش متحرک (Lid-driven cavity)

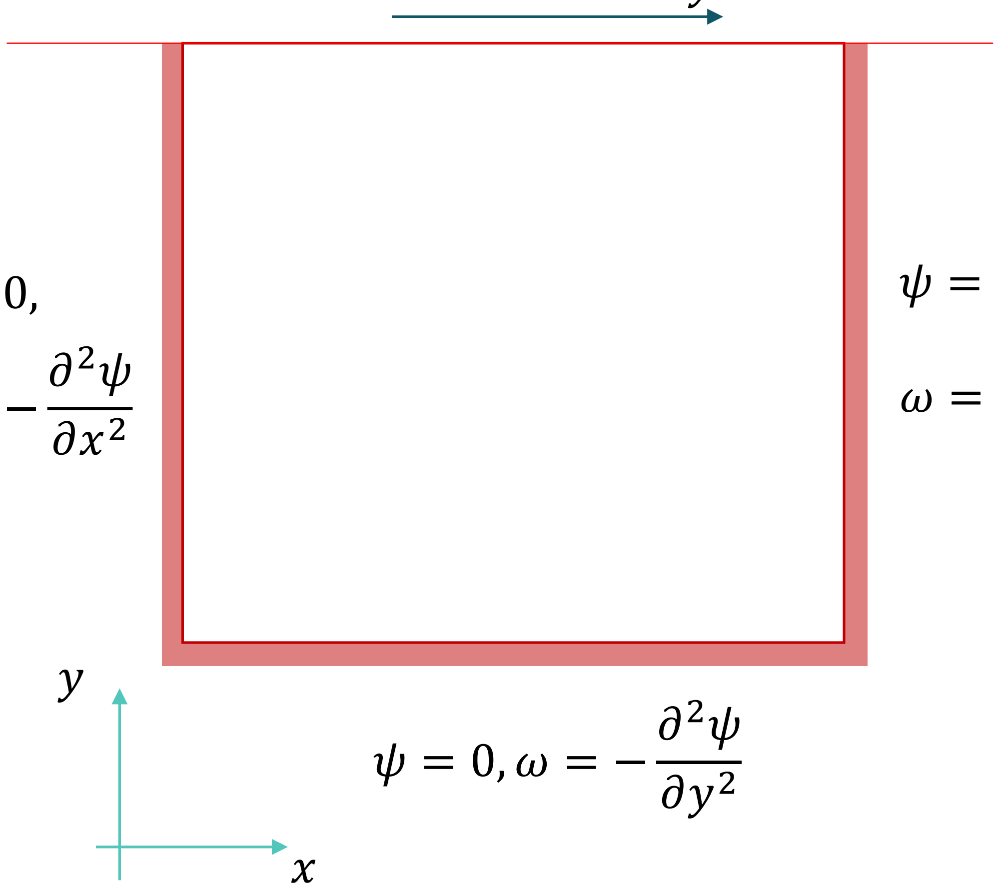
$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=-\omega$$

$$\frac{\partial\omega}{\partial t}=-\frac{\partial\psi}{\partial y}\frac{\partial\omega}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial\omega}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2\omega}{\partial x^2}+\frac{\partial^2\omega}{\partial y^2}\right)$$

$$\omega=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$$

شرایط اولیه

$$\psi=0,\omega=-\frac{\partial^2\psi}{\partial y^2}$$



$$\psi=0,\quad \omega=-\frac{\partial^2\psi}{\partial x^2}$$

$$\psi=0,\quad \omega=-\frac{\partial^2\psi}{\partial x^2}$$

$$\psi=0,\omega=-\frac{\partial^2\psi}{\partial y^2}$$

معادله $\psi - \omega$

گسسته‌سازی معادلات

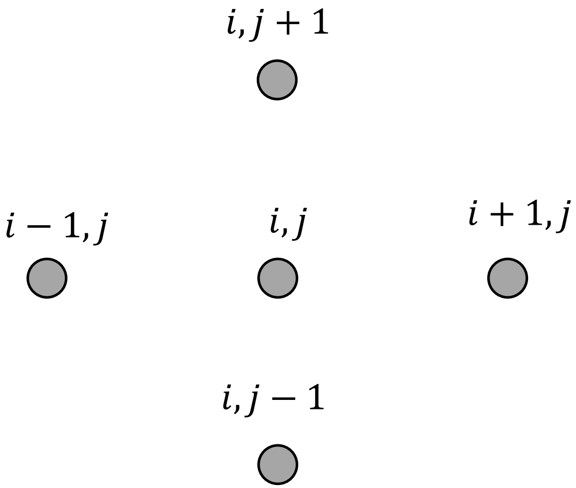
تفاضل محدود

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = -\omega$$

$$\frac{\partial\omega}{\partial t} + \frac{\partial\psi}{\partial y}\frac{\partial\omega}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\omega}{\partial y} = \frac{1}{\mathrm{Re}}\left(\frac{\partial^2\omega}{\partial x^2} + \frac{\partial^2\omega}{\partial y^2}\right)$$

$$\frac{\psi_{i-1,j} - 2\psi_{i,j} + \psi_{i+1,j}}{\Delta x^2} + \frac{\psi_{i,j-1} - 2\psi_{i,j} + \psi_{i,j+1}}{\Delta y^2} = -\omega_{i,j}$$

$$\frac{\mathrm{d}\omega_{i,j}}{\mathrm{d}t} = -\left(\frac{\psi_{i,j+1} - \psi_{i,j-1}}{2\Delta y}\right)\left(\frac{\omega_{i+1,j} - \omega_{i-1,j}}{2\Delta x}\right) + \left(\frac{\psi_{i+1,j} - \psi_{i-1,j}}{2\Delta x}\right)\left(\frac{\omega_{i,j+1} - \omega_{i,j-1}}{2\Delta y}\right) + \frac{1}{Re}\left(\frac{\omega_{i-1,j} - 2\omega_{i,j} + \omega_{i+1,j}}{\Delta x^2} + \frac{\omega_{i,j-1} - 2\omega_{i,j} + \omega_{i,j+1}}{\Delta y^2}\right)$$

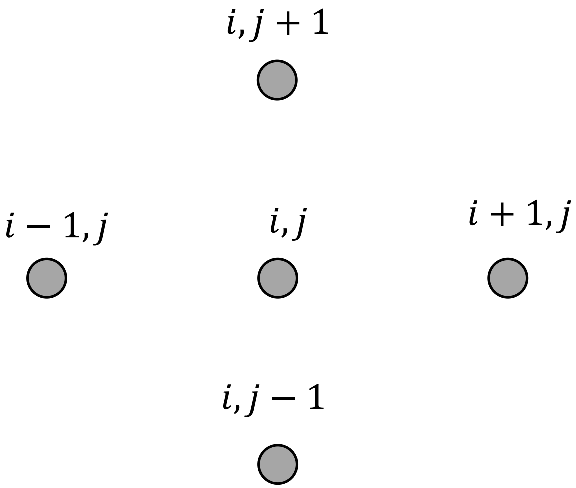


معادله $\psi - \omega$

روش صریح اویلر

$$\frac{\psi_{i-1,j}^{n} - 2\psi_{i,j}^{n} + \psi_{i+1,j}^{n}}{\Delta x^2} + \frac{\psi_{i,j-1}^{n} - 2\psi_{i,j}^{n} + \psi_{i,j+1}^{n}}{\Delta y^2} = -\omega_{i,j}^{n}$$

$$\begin{aligned}\omega_{i,j}^{n+1} = \omega_{i,j}^{n} + \Delta t \Bigg(&-\left(\frac{\psi_{i,j+1}^{n} - \psi_{i,j-1}^{n}}{2\Delta y}\right)\left(\frac{\omega_{i+1,j}^{n} - \omega_{i-1,j}^{n}}{2\Delta x}\right) \\ &+ \left(\frac{\psi_{i+1,j}^{n} - \psi_{i-1,j}^{n}}{2\Delta x}\right)\left(\frac{\omega_{i,j+1}^{n} - \omega_{i,j-1}^{n}}{2\Delta y}\right) \\ &+ \frac{1}{Re}\left(\frac{\omega_{i-1,j}^{n} - 2\omega_{i,j}^{n} + \omega_{i+1,j}^{n}}{\Delta x^2} + \frac{\omega_{i,j-1}^{n} - 2\omega_{i,j}^{n} + \omega_{i,j+1}^{n}}{\Delta y^2}\right)\Bigg)\end{aligned}$$



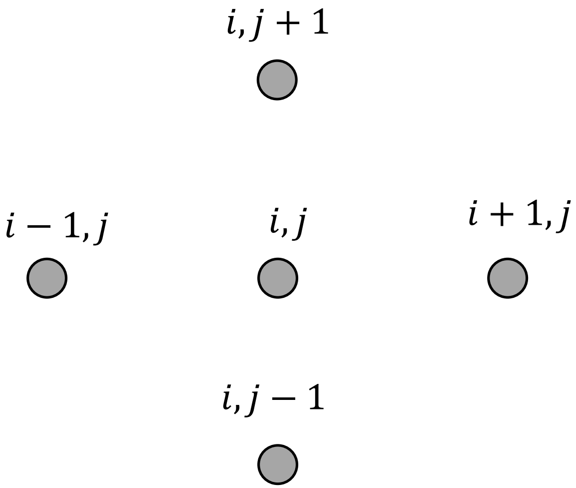
معادله $\psi-\omega$

شرایط اولیه

$$\omega=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$$

$$\omega_{i,j}^{n}=\frac{v_{i+1,j}^{n}-v_{i-1,j}^{n}}{2\Delta x}-\frac{u_{i,j+1}^{n}-u_{i,j-1}^{n}}{2\Delta y}$$

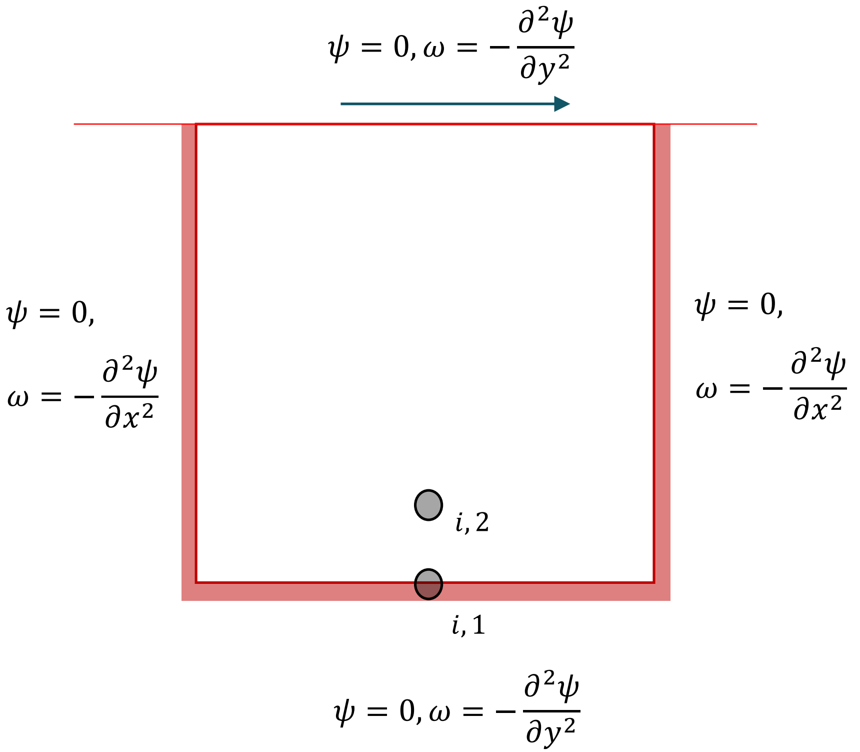
$$\omega_{i,j}^{0}=\frac{v_{i+1,j}^{0}-v_{i-1,j}^{0}}{2\Delta x}-\frac{u_{i,j+1}^{0}-u_{i,j-1}^{0}}{2\Delta y}$$



معادله $\psi - \omega$

گسسته‌سازی معادلات

شرایط مرزی



$:\psi$

$$\boxed{\psi_{i,1} = \psi_{i,M} = \psi_{1,j} = \psi_{N,j} = 0}$$

$:\omega$

مرز جنوبی

$$\psi_{i,2} = \underbrace{\cancel{\psi_{i,1}}}_{0} + \underbrace{\cancel{\frac{\partial \psi}{\partial y}}}_{u=0}\Big|_{i,1} \Delta y + \frac{1}{2}\underbrace{\cancel{\frac{\partial^2 \psi}{\partial y^2}}}_{-\omega_{i,1}}\Big|_{i,1} \Delta y^2$$

$$\psi_{i,2} = \frac{-\omega_{i,1}}{2} \Delta y^2$$

$$\boxed{\omega_{i,1} = -\frac{2\psi_{i,2}}{\Delta y^2}}$$

مرز شمالی

$$\psi_{i,M-1} = \underbrace{\cancel{\psi_{i,M}}}_{0} - \underbrace{\cancel{\frac{\partial \psi}{\partial y}}}_{u=u_w}\Big|_{i,M} \Delta y + \frac{1}{2}\underbrace{\cancel{\frac{\partial^2 \psi}{\partial y^2}}}_{-\omega_{i,M}}\Big|_{i,M} \Delta y^2$$

$$\psi_{i,M-1} = -u_w \Delta y - \frac{\omega_{i,M}}{2} \Delta y^2$$

$$\boxed{\omega_{i,M} = -\frac{2u_w}{\Delta y} - \frac{2\psi_{i,M-1}}{\Delta y^2}}$$

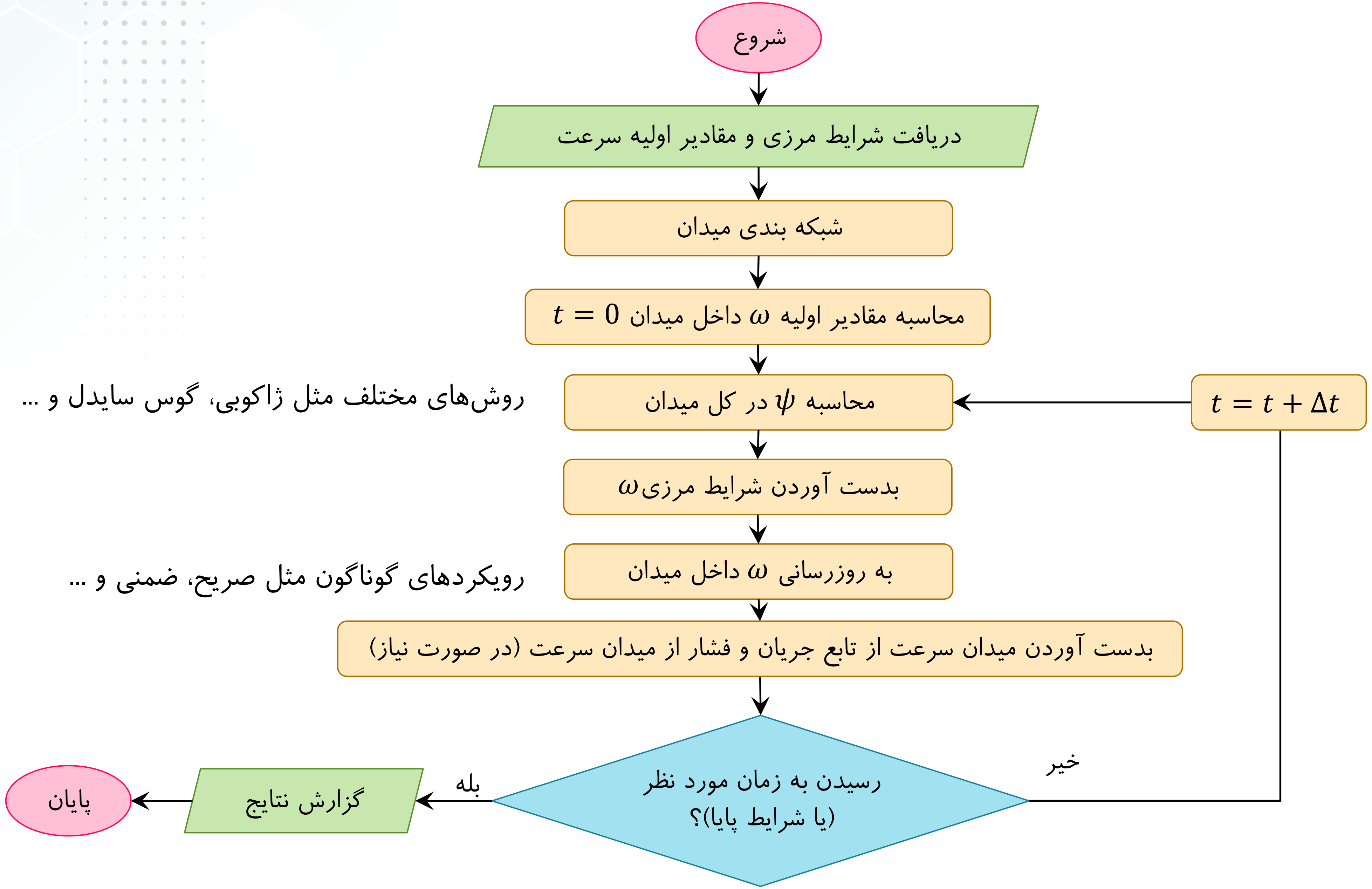
$$\boxed{\omega_{1,j} = -\frac{2\psi_{2,j}}{\Delta x^2}}$$

$$\boxed{\omega_{N,j} = -\frac{2\psi_{N-1,j}}{\Delta x^2}}$$

معادله $\psi - \omega$

گسسته‌سازی معادلات

روند حل



معادله $\psi - \omega$

محاسبه فشار

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)\right)$$

$$\frac{\partial^2 u}{\partial x\partial t}+\frac{\partial u}{\partial x}\frac{\partial u}{\partial x}+u\frac{\partial^2 u}{\partial x^2}+\frac{\partial v}{\partial x}\frac{\partial u}{\partial y}+v\frac{\partial^2 u}{\partial x\partial y}=-\frac{\partial^2 p}{\partial x^2}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^3 u}{\partial x^3}+\frac{\partial^3 u}{\partial x\partial y^2}\right)$$

$$\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)\right)$$

$$\frac{\partial^2 v}{\partial y\partial t}+\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}+u\frac{\partial^2 v}{\partial y\partial x}+\frac{\partial v}{\partial y}\frac{\partial v}{\partial y}+v\frac{\partial^2 v}{\partial y^2}=-\frac{\partial^2 p}{\partial y^2}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^3 v}{\partial y\partial x^2}+\frac{\partial^3 v}{\partial y^3}\right)$$

معادله $\psi - \omega$

محاسبه فشار

$$\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)-\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}+\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}+u\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\frac{\partial v}{\partial x}\frac{\partial u}{\partial y}-\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}+v\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=$$

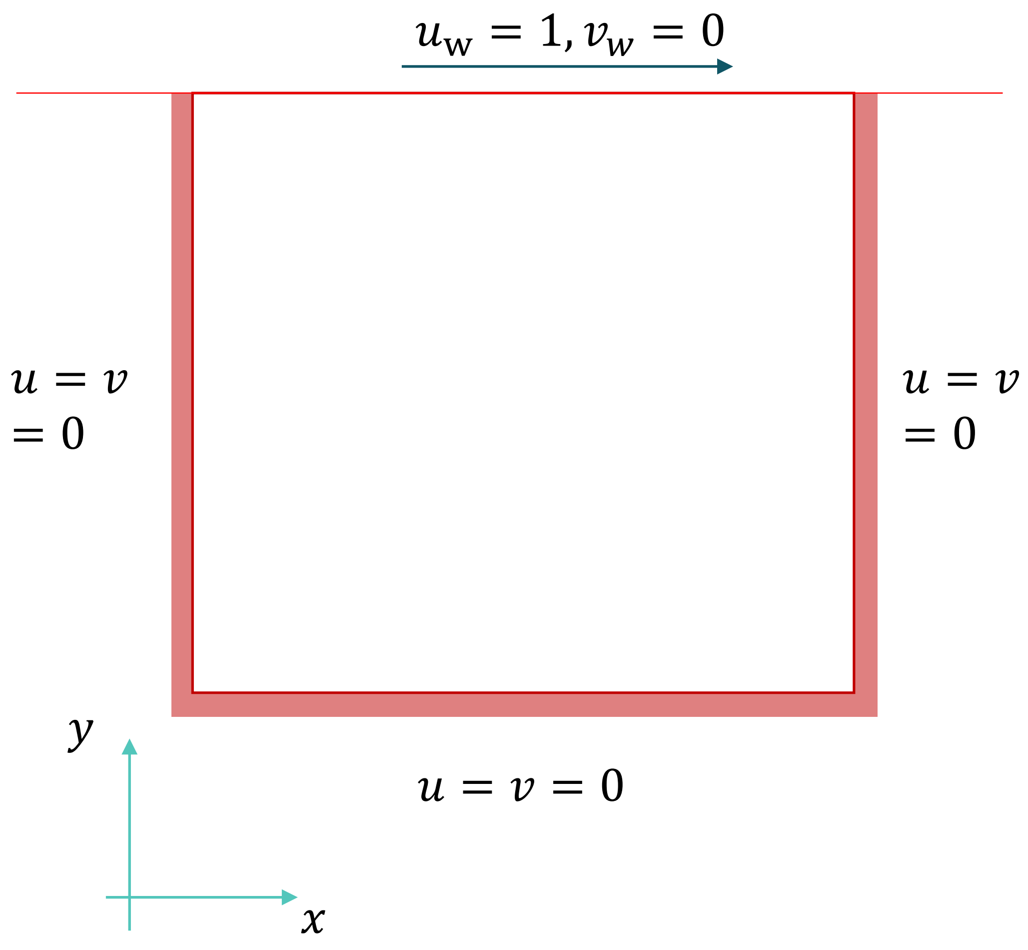
$$-\frac{\partial^2 p}{\partial x^2}-\frac{\partial^2 p}{\partial y^2}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2}{\partial x^2}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\frac{\partial^2}{\partial y^2}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right)$$

$$-\frac{\partial^2 p}{\partial x^2}-\frac{\partial^2 p}{\partial y^2}=2\left(\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}-\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}\right)$$

$$\boxed{\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}=2\left(\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}-\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}\right)}$$

شرایط مرزی

مسئله حفره درپوش متحرک (Lid-driven cavity)



شرط مرزی فشار

دیواره غربی و شرقی

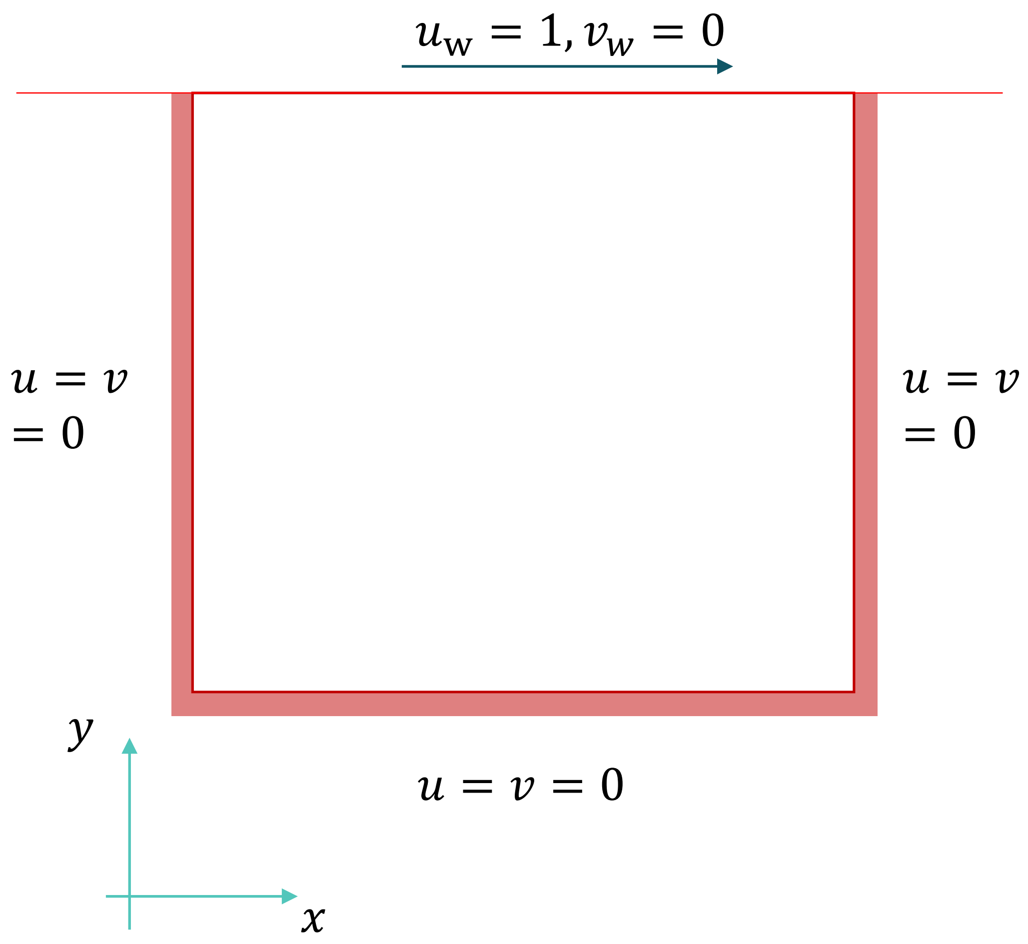
$$\cancelto{0}{\frac{\partial u}{\partial t}} + \cancelto{0}{u\frac{\partial u}{\partial x}} + \cancelto{0}{v\frac{\partial u}{\partial y}} = -\frac{\partial p}{\partial x} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2} + \cancelto{0}{\frac{\partial^2 u}{\partial y^2}}\right)$$

$$\frac{\partial p}{\partial x} = \frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial x^2} \qquad \frac{\partial p}{\partial x} = \frac{1}{\mathrm{Re}}\left(\frac{u_{2,j} - 2u_{1,j} + u_{0,j}}{\Delta x^2}\right)$$

$$\frac{\partial p}{\partial x} = \frac{1}{\mathrm{Re}}\left(\frac{-u_{4,j} + 4u_{3,j} - 5u_{2,j} + 2u_{1,j}}{\Delta x^2}\right)$$

شرایط مرزی

مسئله حفره درپوش متحرک (Lid-driven cavity)



شرط مرزی فشار

دیواره شمالی و جنوبی

$$\cancelto{0}{\frac{\partial v}{\partial t}} + \cancelto{0}{u\frac{\partial v}{\partial x}} + \cancelto{0}{v}\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{\mathrm{Re}}\left(\cancelto{0}{\frac{\partial^2 v}{\partial x^2}} + \frac{\partial^2 v}{\partial y^2}\right)$$

$$\frac{\partial p}{\partial y} = \frac{1}{\mathrm{Re}}\frac{\partial^2 v}{\partial y^2}$$

معادله $\psi - \omega$

گسسته‌سازی

معادله فشار

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 2\left(\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial v}{\partial x}\right)$$

$$\frac{p_{i-1,j}^n - 2p_{i,j}^n + p_{i+1,j}^n}{\Delta x^2} + \frac{p_{i,j-1}^n - 2p_{i,j}^n + p_{i,j+1}^n}{\Delta y^2} =$$

$$2\left(\left(\frac{u_{i+1,j}^n - u_{i-1,j}^n}{2\Delta x}\right)\left(\frac{v_{i,j+1}^n - v_{i,j-1}^n}{2\Delta y}\right) - \left(\frac{u_{i,j+1}^n - u_{i,j-1}^n}{2\Delta y}\right)\left(\frac{v_{i+1,j}^n - v_{i-1,j}^n}{2\Delta x}\right)\right)$$

برای اعمال شرایط مرزی حداقل فشار یک نقطه باید معلوم باشد و یا میدان فشار نسبت به یک نقطه بدست بیاید

معادله $N-S$

معادلات حاکم بر جریان دوبعدی در حالت پایا

continuity: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$x-$momentum: $$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$\frac{\partial(uu)}{\partial x}+\frac{\partial(vu)}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$\frac{\partial(uu)}{\partial x}+\frac{\partial(vu)}{\partial y}=u\frac{\partial u}{\partial x}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+u\frac{\partial v}{\partial y}=u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+u\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}$$

$y-$momentum: $$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

$$\frac{\partial(uv)}{\partial x}+\frac{\partial(vv)}{\partial y}=-\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

معادله $N-S$

معادله جابجایی - پخش

$$\frac{\partial u\phi}{\partial x}+\frac{\partial v\phi}{\partial y}=\alpha\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\right)+S$$

$x-\text{momentum:}$

$$\frac{\partial(uu)}{\partial x}+\frac{\partial(vu)}{\partial y}=\frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\partial p}{\partial x}$$

$$\phi=u \qquad \alpha=\frac{1}{Re} \qquad S=-\frac{\partial p}{\partial x}$$

$y-\text{momentum:}$

$$\frac{\partial(uv)}{\partial x}+\frac{\partial(vv)}{\partial y}=\frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\partial p}{\partial y}$$

$$\phi=v \qquad \alpha=\frac{1}{Re} \qquad S=-\frac{\partial p}{\partial y}$$

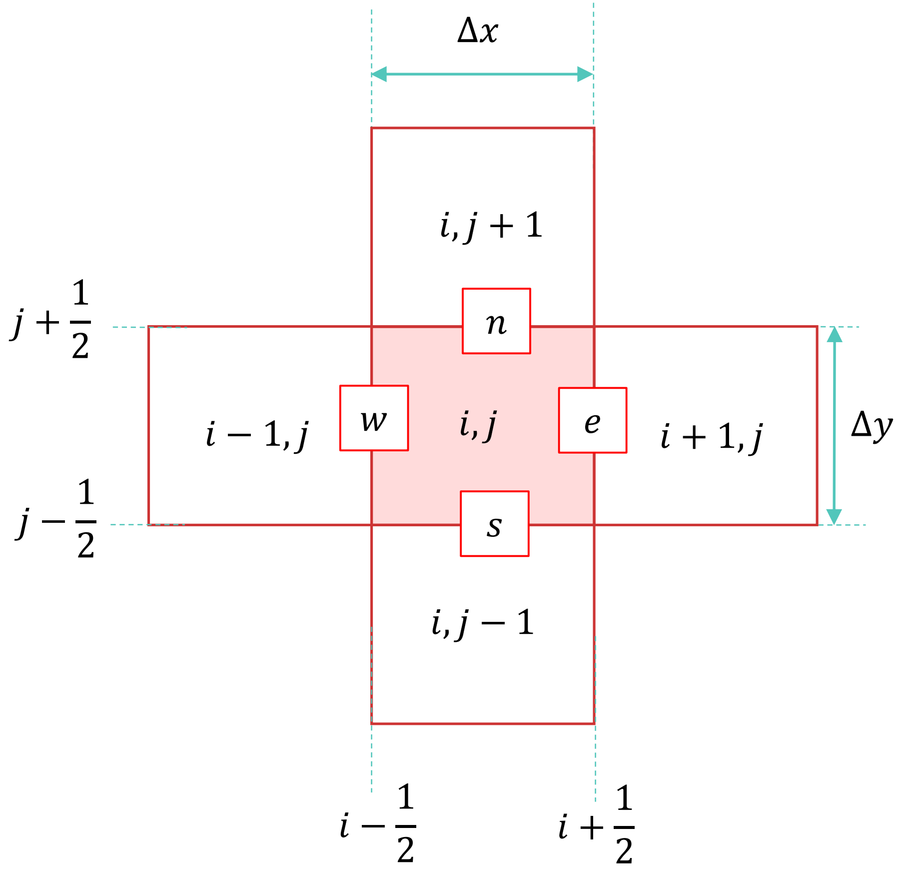
معادله $N - S$

گسسته‌سازی (حجم محدود)

$$\frac{\partial u\phi}{\partial x} + \frac{\partial v\phi}{\partial y} = \alpha\left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}\right) + S$$

$$\frac{u_e\phi_e - u_w\phi_w}{\Delta x} + \frac{v_n\phi_n - v_s\phi_s}{\Delta y} =$$

$$\alpha\left(\frac{\left.\frac{\partial\phi}{\partial x}\right|_e - \left.\frac{\partial\phi}{\partial x}\right|_w}{\Delta x} + \frac{\left.\frac{\partial\phi}{\partial y}\right|_n - \left.\frac{\partial\phi}{\partial y}\right|_s}{\Delta y}\right) + \bar{S}_{i,j}$$



معادله $N-S$

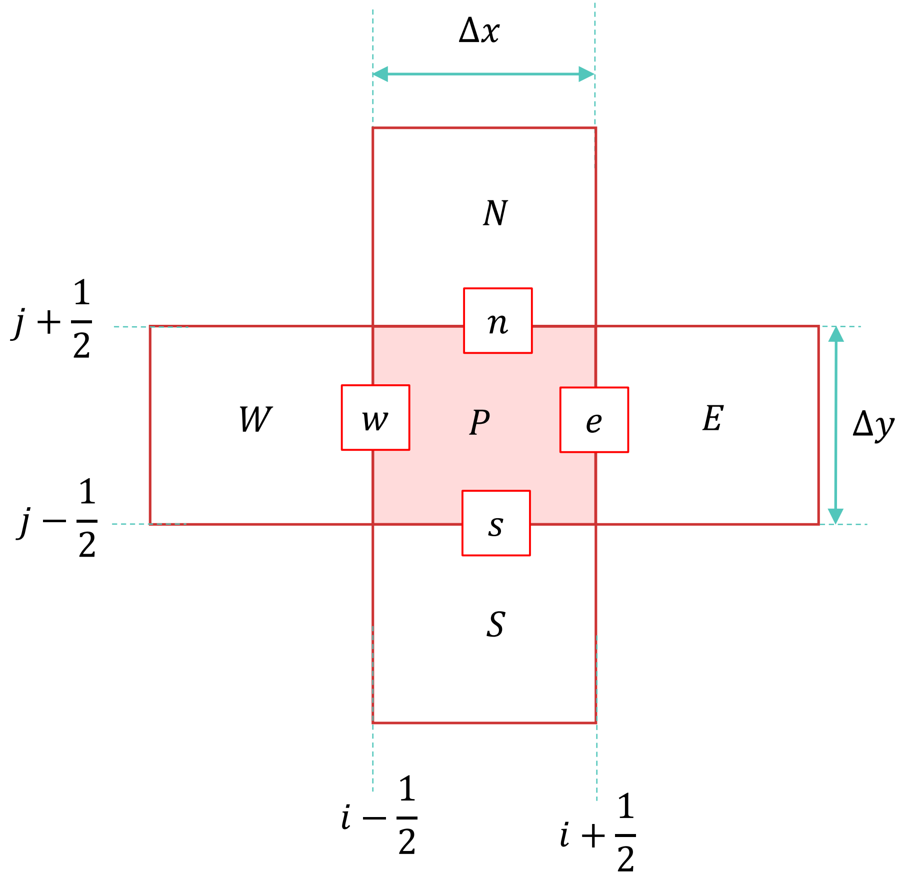
گسسته‌سازی مرکزی

$$u_e = \frac{\bar{u}_{i+1,j} + \bar{u}_{i,j}}{2} + O(\Delta x^2)$$

$$\left.\frac{\partial \phi}{\partial x}\right|_e = \frac{\bar{\phi}_{i+1,j} - \bar{\phi}_{i,j}}{\Delta x} + O(\Delta x^2)$$

$$\phi_e = \frac{\bar{\phi}_{i+1,j} + \bar{\phi}_{i,j}}{2} + O(\Delta x^2)$$

$$\phi_w = \frac{\bar{\phi}_{i,j} + \bar{\phi}_{i-1,j}}{2} + O(\Delta x^2)$$



$$a_P\bar{\phi}_P + a_E\bar{\phi}_E + a_W\bar{\phi}_W + a_N\bar{\phi}_N + a_S\bar{\phi}_S = \bar{S}_P$$

$$a_P\bar{\phi}_P + \sum a_{NB}\bar{\phi}_{NB} = \bar{S}_P$$

معادله $N-S$

$$\frac{\partial u\phi}{\partial x}+\frac{\partial v\phi}{\partial y}=\alpha\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\right)+S$$

$$a_P\bar{\phi}_P+\sum a_{NB}\bar{\phi}_{NB}=\bar{S}_P$$

$x-\text{momentum}:$

$$\frac{\partial(uu)}{\partial x}+\frac{\partial(vu)}{\partial y}=\frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\partial p}{\partial x}$$

$$\phi=u \qquad \alpha=\frac{1}{Re} \qquad S=-\frac{\partial p}{\partial x}$$

$$a_P^u\bar{u}_P+\sum a_{NB}^u\bar{u}_{NB}=\bar{S}_P^u$$

$y-\text{momentum}:$

$$\frac{\partial(uv)}{\partial x}+\frac{\partial(vv)}{\partial y}=\frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\partial p}{\partial y}$$

$$a_P^v\bar{v}_P+\sum a_{NB}^v\bar{v}_{NB}=\bar{S}_P^v$$

$$\phi=v \qquad \alpha=\frac{1}{Re} \qquad S=-\frac{\partial p}{\partial y}$$

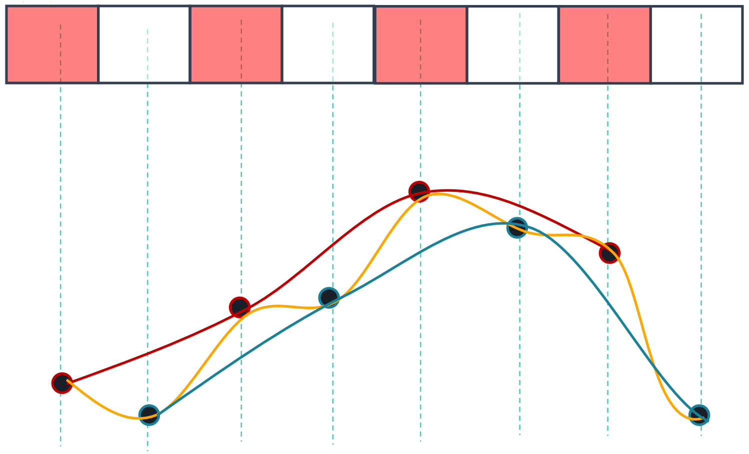
معادله $N-S$

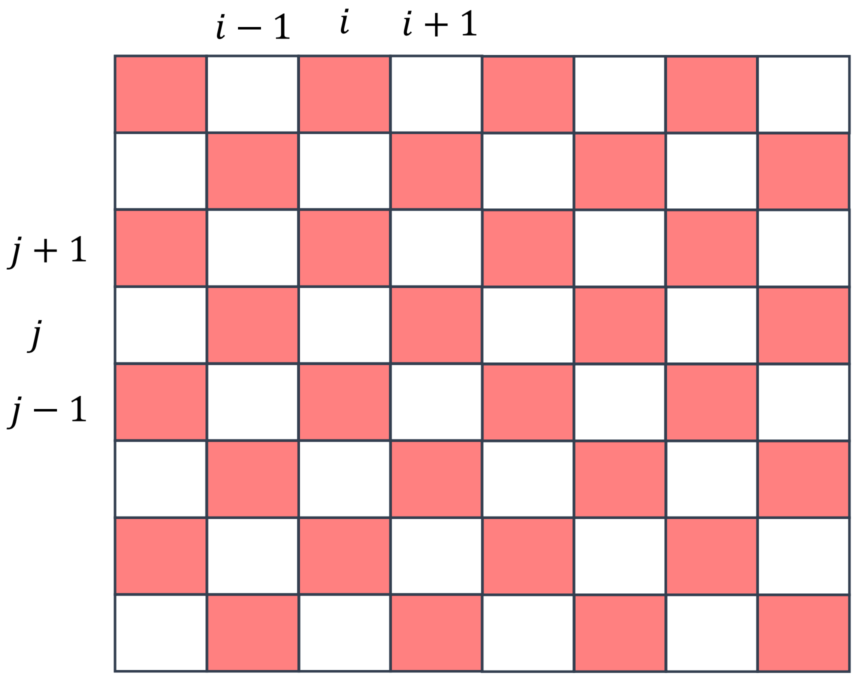
گسسته‌سازی (حجم محدود)

گسسته‌سازی چشمه (گرادیان فشار)

$$-\bar{S}_P^u = \left.\frac{\partial p}{\partial x}\right|_{i,j} = \frac{p_{i+1,j} - p_{i-1,j}}{2\Delta x}$$

$$-\bar{S}_P^v = \left.\frac{\partial p}{\partial y}\right|_{i,j} = \frac{p_{i,j+1} - p_{i,j-1}}{2\Delta y}$$

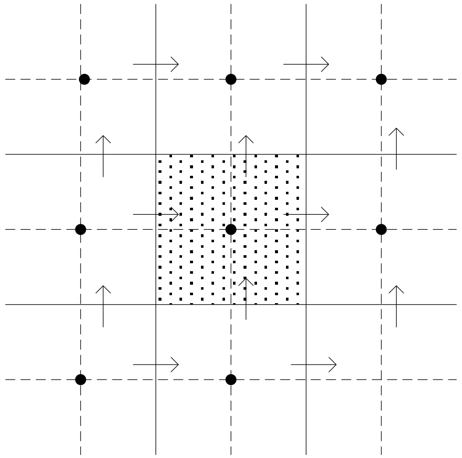




میدان فشار صفحه شطرنجی (Checkerboard pressure field)

روش شبکه متناوب (Staggered grid)

p:

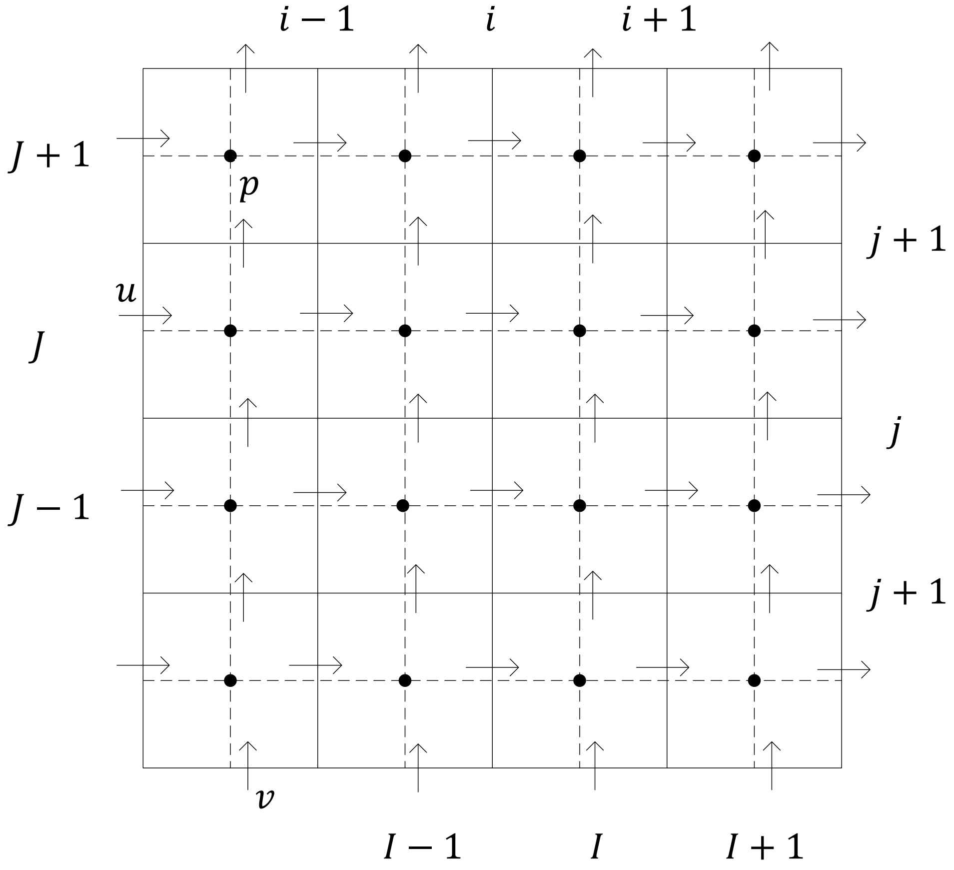


continuity:

$p_{I,J}$

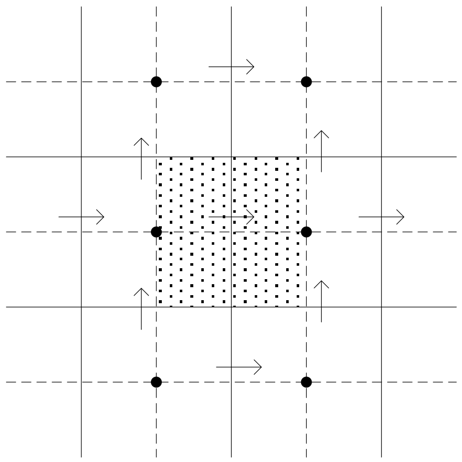
$I = 1, 2, \cdots, N$

$J = 1, 2, \cdots, M$

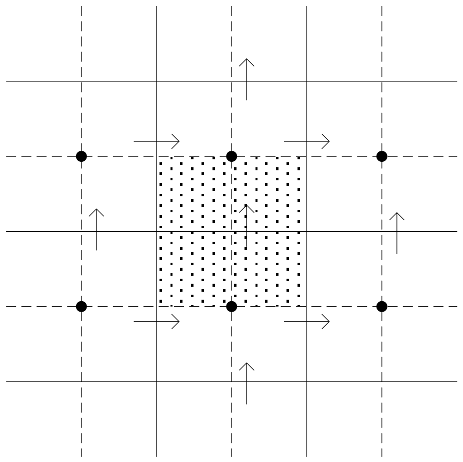


روش شبکه متناوب (Staggered grid)

u:



v:



$x-\text{momentum}$:

$u_{i,J}$

$i = 1, 2, \cdots, N+1$

$J = 1, 2, \cdots, M$

$y-\text{momentum}$:

$v_{I,j}$

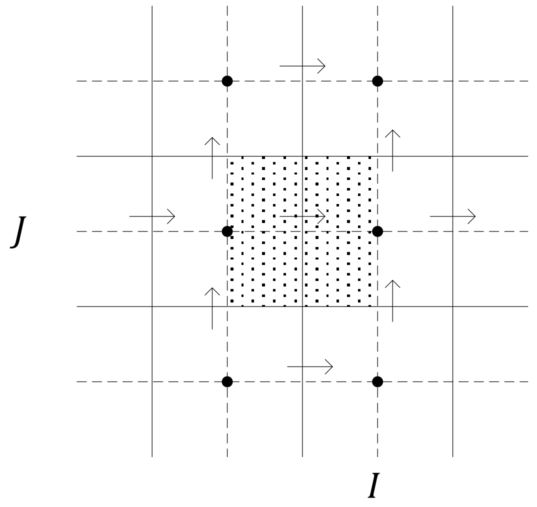
$I = 1, 2, \cdots, N$

$j = 1, 2, \cdots, M+1$

معادله $N-S$

گسسته‌سازی (حجم محدود)

معادله مومنتوم در راستای x



$$u_e = \frac{u_{i,J} + u_{i+1,J}}{2} \qquad u_w = \frac{u_{i-1,J} + u_{i,J}}{2}$$

$$v_n = \frac{v_{I-1,j+1} + v_{I,j+1}}{2} \qquad v_s = \frac{v_{I-1,j} + v_{I,j}}{2}$$

$$\left.\frac{\partial u}{\partial x}\right|_e = \frac{u_{i+1,J} - u_{i,J}}{\Delta x} \qquad \left.\frac{\partial u}{\partial x}\right|_w = \frac{u_{i,J} - u_{i-1,J}}{\Delta x}$$

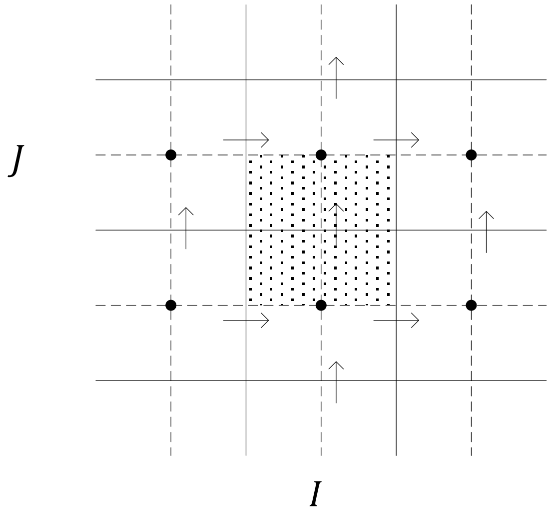
$$\left.\frac{\partial u}{\partial y}\right|_n = \frac{u_{i,J+1} - u_{i,J}}{\Delta y} \qquad \left.\frac{\partial u}{\partial y}\right|_s = \frac{u_{i,J} - u_{i,\mathrm{J}-1}}{\Delta y}$$

$$\bar{S}_P^u = -\left.\frac{\partial p}{\partial x}\right|_{i,J} = \frac{-p_{I,J} + p_{I-1,J}}{\Delta x} + O(\Delta x^2)$$

$$a_{i,J}^u u_{i,J} + \sum a_{nB}^u u_{nB} = S_{i,J}^u$$

معادله $N-S$

معادله مومنتوم در راستای y



$$u_e = \frac{u_{i+1,J} + u_{i+1,J-1}}{2} \qquad u_w = \frac{u_{i,J} + u_{i,J-1}}{2}$$

$$v_n = \frac{v_{I,j} + v_{I,j+1}}{2} \qquad v_s = \frac{v_{I,j} + v_{I,j-1}}{2}$$

$$\left.\frac{\partial v}{\partial x}\right|_e = \frac{v_{I+1,j} - v_{I,j}}{\Delta x} \qquad \left.\frac{\partial v}{\partial x}\right|_w = \frac{v_{I,j} - v_{I-1,j}}{\Delta x}$$

$$\left.\frac{\partial v}{\partial y}\right|_n = \frac{v_{I,j+1} - v_{I,j}}{\Delta y} \qquad \left.\frac{\partial v}{\partial y}\right|_s = \frac{v_{I,j} - v_{I,j-1}}{\Delta y}$$

$$a_{I,j}^v v_{I,j} + \sum a_{Nb}^v v_{Nb} = S_{I,j}^v$$

$$\bar{S}_P^v = -\left.\frac{\partial p}{\partial y}\right|_{i,j} = \frac{-p_{I,J} + p_{I,J-1}}{\Delta y} + O(\Delta y^2)$$

چالش

continuity:

فاقد فشار

$x-$ momentum:

$$\frac{\partial p}{\partial x}$$

$y-$ momentum:

$$\frac{\partial p}{\partial y}$$

راه حل

Semi − Implicit Method for Pressure − Linked Equation (SIMPLE)

۱. حدس u^*, v^* $\leftarrow$ محاسبه $\left.\begin{matrix} a^u_{i,J}, a^u_{nB} \\ a^v_{I,j}, a^v_{Nb} \end{matrix}\right\}$

۲. حدس p^* $(p^*=0)$ $\leftarrow$ حل دستگاه

$$\left.\begin{aligned} a^u_{i,J}u_{i,J} + \sum a^u_{nB}u_{nB} &= \frac{-p_{I,J}+p_{I-1,J}}{\Delta x} \\ a^v_{I,j}v_{I,j} + \sum a^v_{Nb}v_{Nb} &= \frac{-p_{I,J}+p_{I,J-1}}{\Delta y} \end{aligned}\right\}$$

الگوریتم SIMPLE

$$p = p^* + p' \qquad u = u^* + u' \qquad v = v^* + v'$$

$$p' = p - p^* \qquad u' = u - u^* \qquad v' = v - v^*$$

$$\begin{cases} a^u_{i,J} u_{i,J} + \sum a^u_{nB} u_{nB} = \dfrac{-p_{I,J} + p_{I-1,J}}{\Delta x} \\ a^u_{i,J} u^*_{i,J} + \sum a^u_{nB} u^*_{nB} = \dfrac{-p^*_{I,J} + p^*_{I-1,J}}{\Delta x} \end{cases}$$

$$\begin{cases} a^u_{i,J} u'_{i,J} + \sum a^u_{nB} u'_{nB} = \dfrac{-p'_{I,J} + p'_{I-1,J}}{\Delta x} \\ a^v_{I,j} v'_{I,j} + \sum a^v_{Nb} v'_{Nb} = \dfrac{-p'_{I,J} + p'_{I,J-1}}{\Delta y} \end{cases}$$

$$\begin{cases} a^v_{I,j} v_{I,j} + \sum a^u_{Nb} v_{Nb} = \dfrac{-p_{I,J} + p_{I,J-1}}{\Delta y} \\ a^v_{I,j} v^*_{I,j} + \sum a^v_{Nb} v^*_{Nb} = \dfrac{-p^*_{I,J} + p^*_{I,J-1}}{\Delta y} \end{cases}$$

۱. حدس u^*, v^* $\leftarrow$ محاسبه $\left.\begin{matrix} a_{i,J}^u, a_{nB}^u \\ a_{I,j}^v, a_{Nb}^u \end{matrix}\right\}$

۲. حدس p^* $(p^*=0)$ $\leftarrow$ بروز رسانی u^*, v^* با استفاده از روش‌های ADI, SOR, …

۳. محاسبه p' ؟؟؟

۴. محاسبه u', v'

۵. حل کامل مشخصات جریان

$$p = p^* + p'$$
$$u = u^* + u'$$
$$v = v^* + v'$$

معادله $N-S$

بدست آوردن فشار

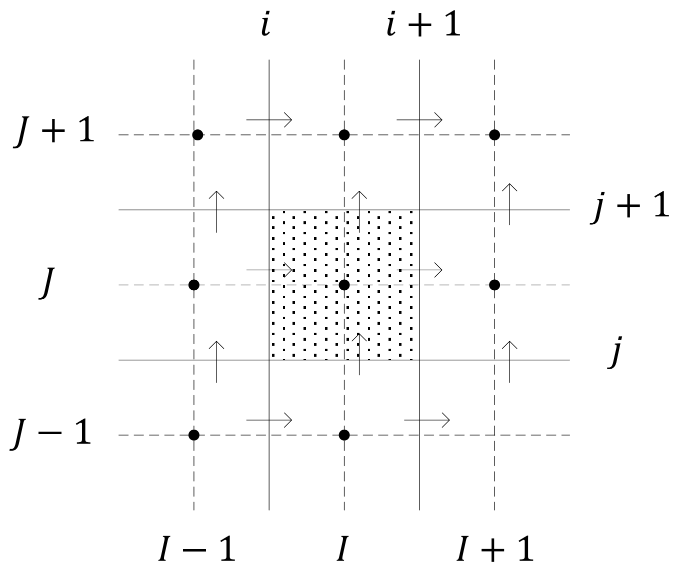
معادله پیوستگی

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\frac{1}{A}\int(\nabla . V)\mathrm{d}\,A=0$$

$$\frac{1}{A}\int(\vec{V}.\hat{n})\mathrm{d}\,l=0$$

$$\frac{u_e-u_w}{\Delta x}+\frac{v_n-v_s}{\Delta y}=0$$



معادله $N-S$

الگوریتم SIMPLE

بدست آوردن فشار

$$\frac{u_{i+1,J}-u_{i,J}}{\Delta x}+\frac{v_{I,j+1}-v_{I,j}}{\Delta y}=0$$

$$u_{i,J}=u_{i,J}^{*}+u_{i,J}'$$

$$v_{I,j}=v_{I,j}^{*}+v_{I,j}'$$

$$\frac{\left(u_{i+1,J}^{*}+u_{i+1,J}'\right)-\left(u_{i,J}^{*}+u_{i,J}'\right)}{\Delta x}+\frac{\left(v_{I,j+1}^{*}+v_{I,j+1}'\right)-\left(v_{I,j}^{*}+v_{I,j}'\right)}{\Delta y}=0$$

$$\frac{u_{i+1,J}'-u_{i,J}'}{\Delta x}+\frac{v_{I,j+1}'-v_{I,j}'}{\Delta y}=\frac{u_{i+1,J}^{*}-u_{i,J}^{*}}{\Delta x}+\frac{v_{I,j+1}^{*}-v_{I,j}^{*}}{\Delta y}$$

$$\frac{u_{i+1,J}'-u_{i,J}'}{\Delta x}+\frac{v_{I,j+1}'-v_{I,j}'}{\Delta y}=\mathrm{Res}_{I,J}$$

معادله $N-S$

الگوریتم SIMPLE

بدست آوردن فشار

$$\mathrm{Res}_{I,J} = \frac{u^*_{i+1,J}-u^*_{i,J}}{\Delta x} + \frac{v^*_{I,j+1}-v^*_{I,j}}{\Delta y}$$

$$\frac{u'_{i+1,J}-u'_{i,J}}{\Delta x} + \frac{v'_{I,j+1}-v'_{I,j}}{\Delta y} = \mathrm{Res}_{I,J}$$

$$\begin{cases} a^u_{i,J}u'_{i,J} + \underbrace{\sum a^u_{nB}u'_{nB}}_{0} = \dfrac{-p'_{I,J}+p'_{I-1,J}}{\Delta x} \\ a^v_{I,j}v'_{I,j} + \underbrace{\sum a^u_{Nb}v'_{Nb}}_{0} = \dfrac{-p'_{I,J}+p'_{I,J-1}}{\Delta y} \end{cases} \rightarrow \begin{cases} u'_{i,J} = \dfrac{-p'_{I,J}+p'_{I-1,J}}{a^u_{i,J}\Delta x} \\ v'_{I,j} = \dfrac{-p'_{I,J}+p'_{I,J-1}}{a^v_{I,j}\Delta y} \end{cases}$$

فرض اساسی روش SIMPLE

$$\begin{cases} \sum a^u_{nB}u'_{nB} = 0 \\ \sum a^u_{Nb}v'_{Nb} = 0 \end{cases}$$

معادله $N-S$

الگوریتم SIMPLE

بدست آوردن فشار

$$\frac{u'_{i+1,J}-u'_{i,J}}{\Delta x}+\frac{v'_{I,j+1}-v'_{I,j}}{\Delta y}=\mathrm{Res}_{I,J}$$

$$\frac{\left(\dfrac{-p'_{I+1,J}+p'_{I,J}}{a^u_{i+1,J}\Delta x}\right)-\left(\dfrac{-p'_{I,J}+p'_{I-1,J}}{a^u_{i,J}\Delta x}\right)}{\Delta x}+\frac{\left(\dfrac{-p'_{I,J+1}+p'_{I,J}}{a^v_{I,j+1}\Delta y}\right)-\left(\dfrac{-p'_{I,J}+p'_{I,J-1}}{a^v_{I,j}\Delta y}\right)}{\Delta y}=\mathrm{Res}_{I,J}$$

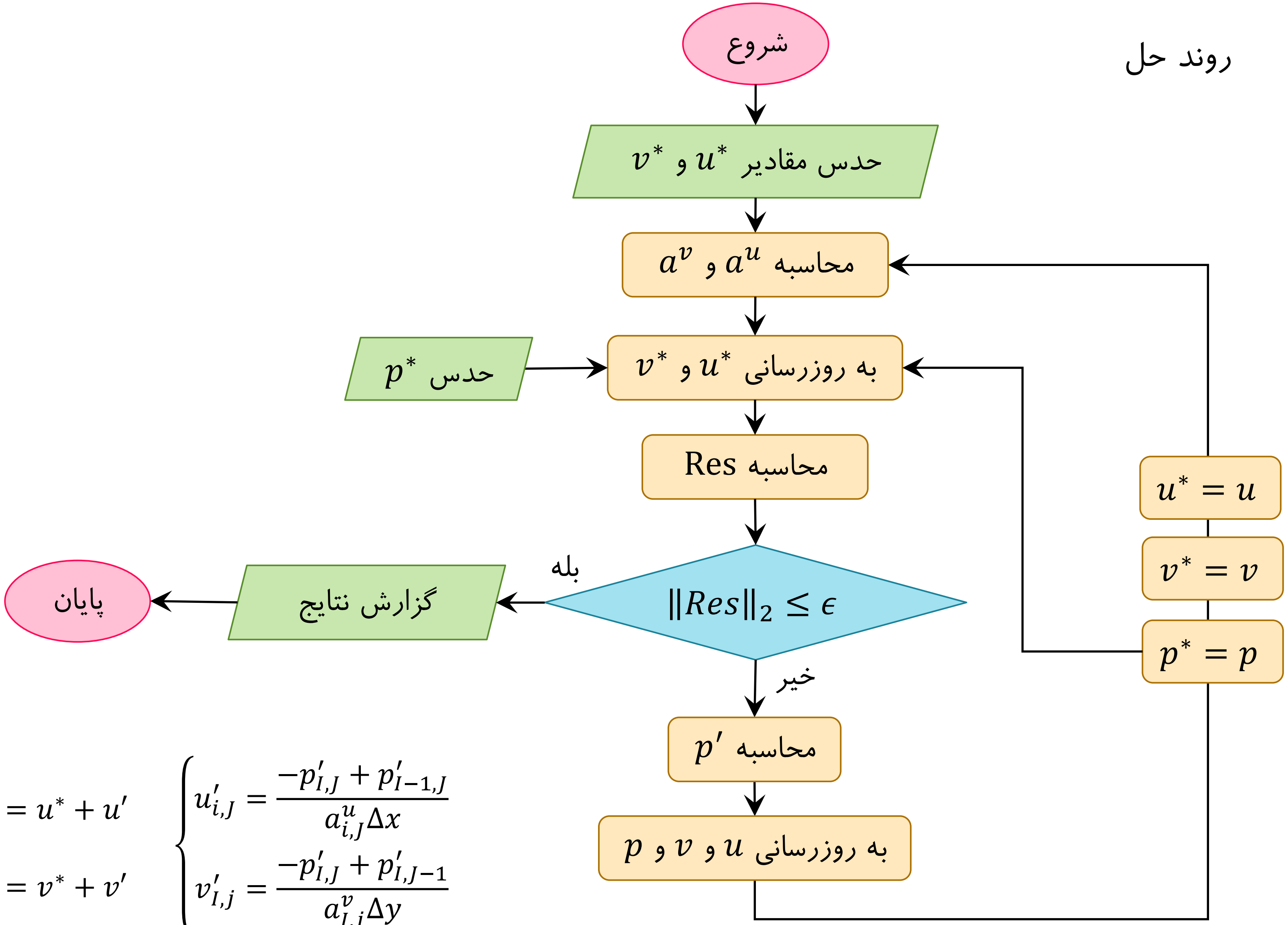
$$a^p_{I,J}p'_{I,J}+a^p_{I+1,J}p'_{I+1,J}+a^p_{I-1,J}p'_{I-1,J}+a^p_{I,J+1}p'_{I,J+1}+a^p_{I,J-1}p'_{I,J-1}=\mathrm{Res}_{I,J}$$

$$a^p_{I,J}p'_{I,J}+\sum a^p_{NB}p'_{NB}=\mathrm{Res}_{I,J}$$

معادله $N-S$

الگوریتم SIMPLE

روند حل



$$\begin{cases} a_{i,J}^u u_{i,J}^* + \sum a_{nB}^u u_{nB}^* = \dfrac{-p_{I,J}^* + p_{I-1,J}^*}{\Delta x} \\ a_{I,j}^v v_{I,j}^* + \sum a_{Nb}^v v_{Nb}^* = \dfrac{-p_{I,J}^* + p_{I,J-1}^*}{\Delta y} \end{cases}$$

$$\mathrm{Res}_{I,J} = \frac{u_{i+1,J}^* - u_{i,J}^*}{\Delta x} + \frac{v_{I,j+1}^* - v_{I,j}^*}{\Delta y}$$

$$a_{I,J}^p p_{I,J}' + \sum a_{NB}^p p_{NB}' = \mathrm{Res}_{I,J}$$

$$p = p^* + p'$$

$$u = u^* + u'$$

$$v = v^* + v'$$

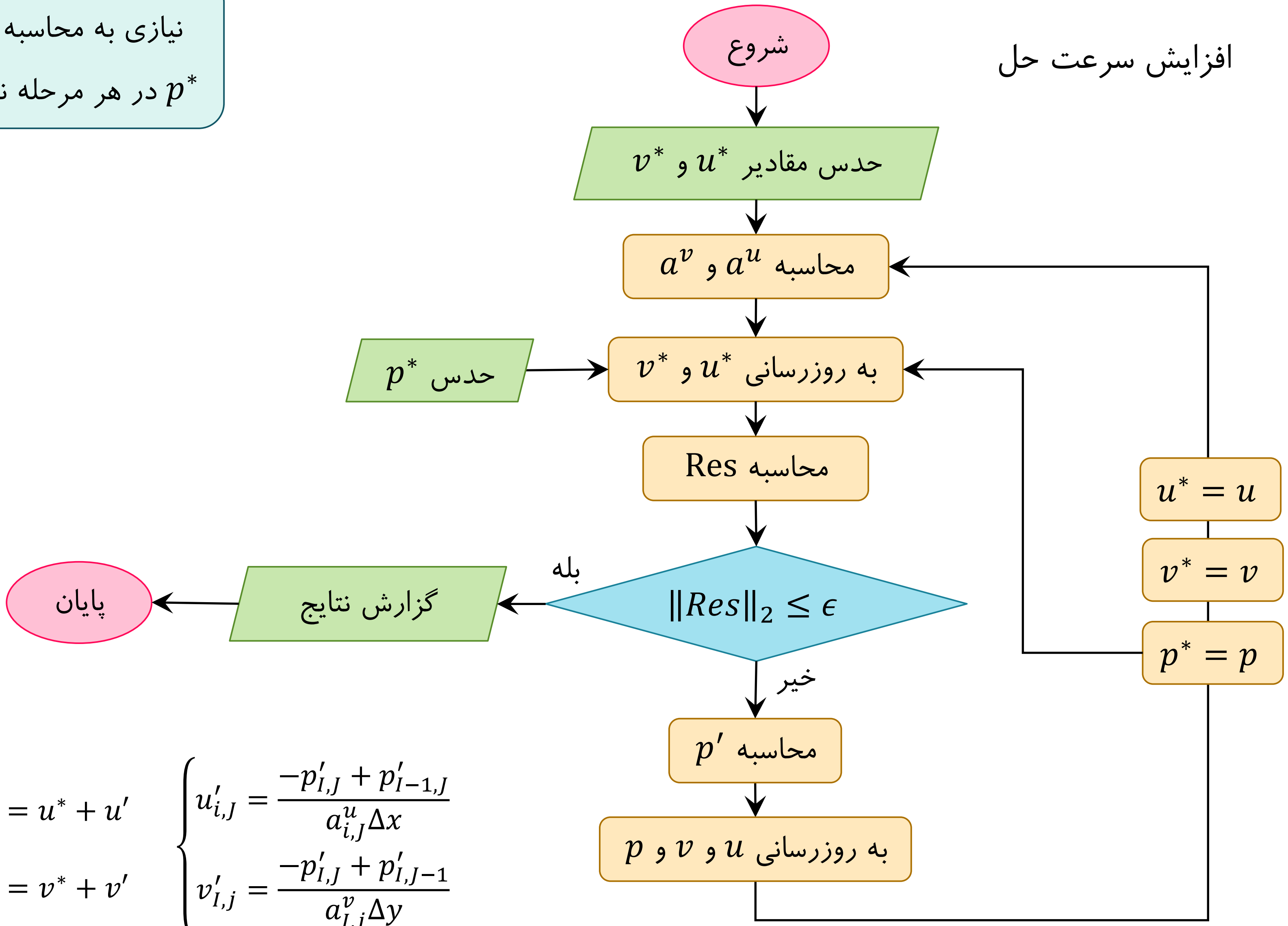
$$\begin{cases} u_{i,J}' = \dfrac{-p_{I,J}' + p_{I-1,J}'}{a_{i,J}^u \Delta x} \\ v_{I,j}' = \dfrac{-p_{I,J}' + p_{I,J-1}'}{a_{I,j}^v \Delta y} \end{cases}$$

معادله $N-S$

الگوریتم SIMPLE

افزایش سرعت حل

نیازی به محاسبه دقیق مقادیر u^* و v^* و همچنین p^* در هر مرحله نیست و یک یا چند گام کافی است.



$$\begin{cases} u^*_{i,J} = -\dfrac{\sum a^u_{nB} u^*_{nB}}{a^u_{i,J}} + \dfrac{-p^*_{I,J} + p^*_{I-1,J}}{a^u_{i,J}\Delta x} \\ v^*_{I,j} = -\dfrac{\sum a^u_{Nb} v^*_{Nb}}{a^v_{I,j}} + \dfrac{-p^*_{I,J} + p^*_{I,J-1}}{a^v_{I,j}\Delta y} \end{cases}$$

$$\mathrm{Res}_{I,J} = \frac{u^*_{i+1,J} - u^*_{i,J}}{\Delta x} + \frac{v^*_{I,j+1} - v^*_{I,j}}{\Delta y}$$

$$p'_{I,J} = -\frac{\sum a^p_{NB} p'_{NB} + \mathrm{Res}_{I,J}}{a^p_{I,J}}$$

$$p = p^* + p'$$

$$u = u^* + u'$$

$$v = v^* + v'$$

$$\begin{cases} u'_{i,J} = \dfrac{-p'_{I,J} + p'_{I-1,J}}{a^u_{i,J}\Delta x} \\ v'_{I,j} = \dfrac{-p'_{I,J} + p'_{I,J-1}}{a^v_{I,j}\Delta y} \end{cases}$$

الگوریتم SIMPLE

Under – relaxation factor (URF)

$$0 \le \alpha \le 1$$

افزایش پایداری

$$\begin{cases} u_{i,J}^{*} = -\dfrac{\sum a_{nB}^{u} u_{nB}^{*}}{a_{i,J}^{u}} + \dfrac{-p_{I,J}^{*} + p_{I-1,J}^{*}}{a_{i,J}^{u}\Delta x} \\ v_{I,j}^{*} = -\dfrac{\sum a_{Nb}^{u} v_{Nb}^{*}}{a_{I,j}^{v}} + \dfrac{-p_{I,J}^{*} + p_{I,J-1}^{*}}{a_{I,j}^{v}\Delta y} \end{cases}$$

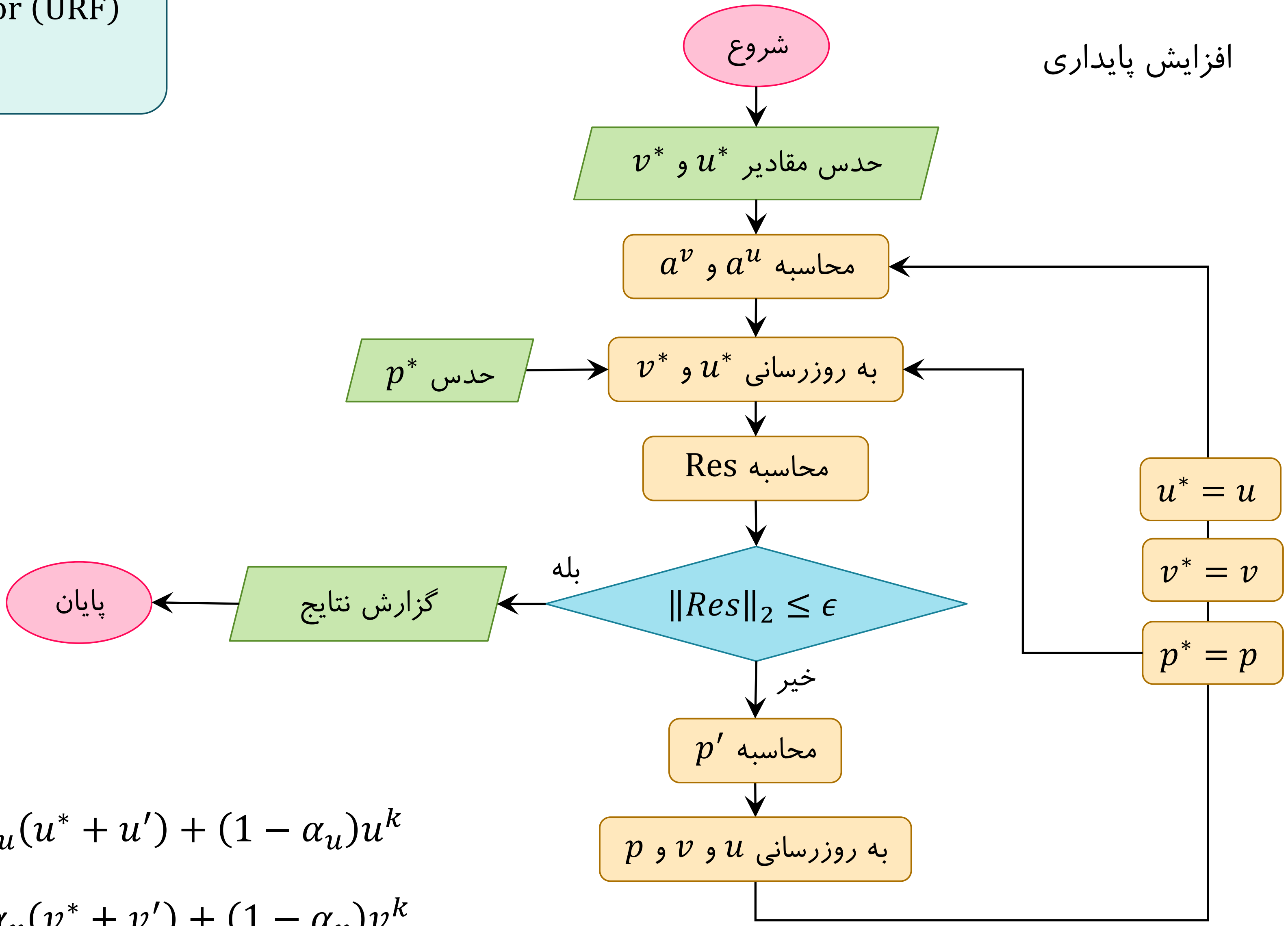
$$\mathrm{Res}_{I,J} = \frac{u_{i+1,J}^{*} - u_{i,J}^{*}}{\Delta x} + \frac{v_{I,j+1}^{*} - v_{I,j}^{*}}{\Delta y}$$

$$p'_{I,J} = -\frac{\sum a_{NB}^{p} p'_{NB} + \mathrm{Res}_{I,J}}{a_{I,J}^{p}}$$

$$p = p^{*} + \alpha_p p'$$

$$u^{k+1} = \alpha_u (u^{*} + u') + (1-\alpha_u) u^{k}$$

$$v^{k+1} = \alpha_v (v^{*} + v') + (1-\alpha_v) v^{k}$$



الگوریتم SIMPLEC

$$\begin{cases} u'_{nB} = -u'_{i,J} \\ v'_{Nb} = -v'_{I,j} \end{cases} \rightarrow \begin{cases} u'_{i,J} = \dfrac{-p'_{I,J} + p'_{I-1,J}}{\left(a^u_{i,J} - \sum a^u_{nB}\right)\Delta x} \\ v'_{I,j} = \dfrac{-p'_{I,J} + p'_{I,J-1}}{\left(a^v_{I,j} - \sum a^u_{Nb}\right)\Delta y} \end{cases}$$

$$\begin{cases} u^*_{i,J} = -\dfrac{\sum a^u_{nB} u^*_{nB}}{a^u_{i,J}} + \dfrac{-p^*_{I,J} + p^*_{I-1,J}}{a^u_{i,J}\Delta x} \\ v^*_{I,j} = -\dfrac{\sum a^u_{Nb} v^*_{Nb}}{a^v_{I,j}} + \dfrac{-p^*_{I,J} + p^*_{I,J-1}}{a^v_{I,j}\Delta y} \end{cases}$$

$$\mathrm{Res}_{I,J} = \frac{u^*_{i+1,J} - u^*_{i,J}}{\Delta x} + \frac{v^*_{I,j+1} - v^*_{I,j}}{\Delta y}$$

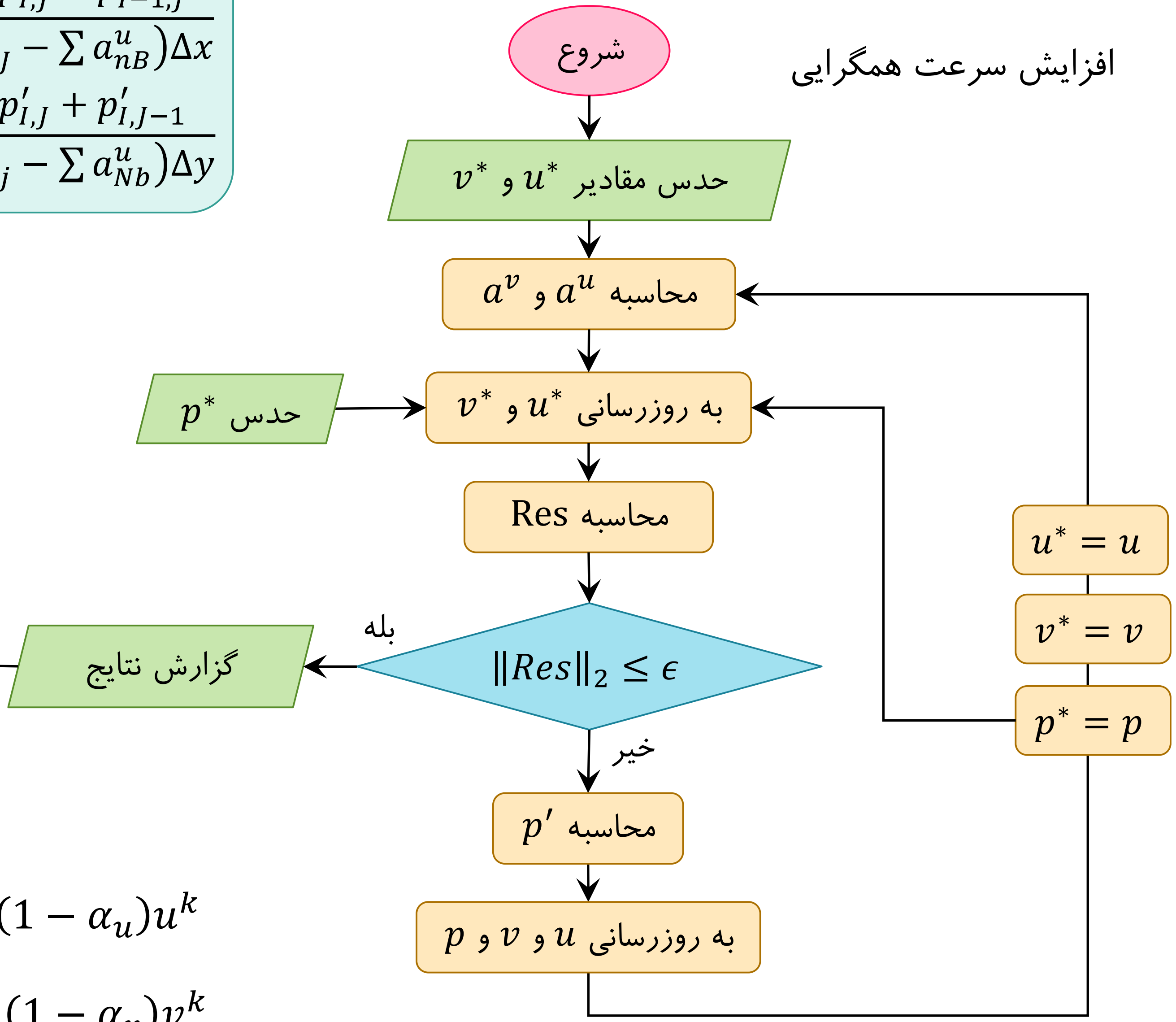
$$p'_{I,J} = -\frac{\sum a^p_{NB} p'_{NB} + \mathrm{Res}_{I,J}}{a^p_{I,J}}$$

$$p = p^* + \alpha_p p'$$

$$u^{k+1} = \alpha_u(u^* + u') + (1 - \alpha_u)u^k$$

$$v^{k+1} = \alpha_v(v^* + v') + (1 - \alpha_v)v^k$$

افزایش سرعت همگرایی



الگوریتم SIMPLER

افزایش سرعت همگرایی

$$\begin{cases} \hat{u}_{i,J} = -\dfrac{\sum a^u_{nB} u_{nB}}{a^u_{i,J}} \\ \hat{v}_{I,j} = -\dfrac{\sum a^u_{Nb} v_{Nb}}{a^v_{I,j}} \end{cases}$$

$$\begin{cases} u_{i,J} = \hat{u}_{i,J} + \dfrac{-p_{I,J} + p_{I-1,J}}{a^u_{i,J}\Delta x} \\ v_{I,j} = \hat{v}_{I,j} + \dfrac{-p_{I,J} + p_{I,J-1}}{a^v_{I,j}\Delta y} \end{cases}$$

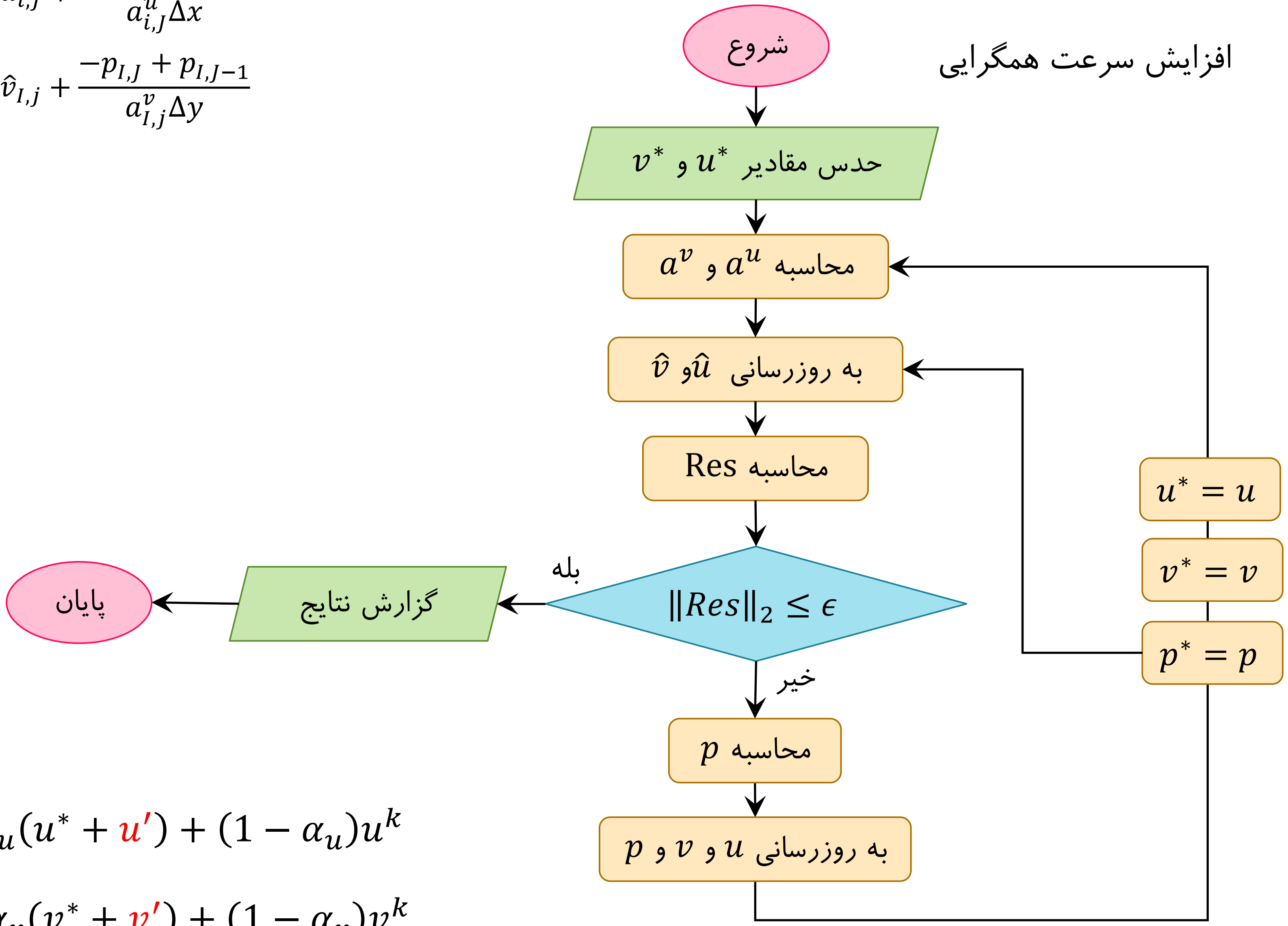
$$\begin{cases} u^*_{i,J} = -\dfrac{\sum a^u_{nB} u^*_{nB}}{a^u_{i,J}} + \dfrac{-p^*_{I,J} + p^*_{I-1,J}}{a^u_{i,J}\Delta x} \\ v^*_{I,j} = -\dfrac{\sum a^u_{Nb} v^*_{Nb}}{a^v_{I,j}} + \dfrac{-p^*_{I,J} + p^*_{I,J-1}}{a^v_{I,j}\Delta y} \end{cases}$$

$$\mathrm{Res}_{I,J} = \frac{\hat{u}_{i+1,J} - \hat{u}_{i,J}}{\Delta x} + \frac{\hat{v}_{I,j+1} - \hat{v}_{I,j}}{\Delta y}$$

$$p_{I,J} = -\frac{\sum a_{NB} p'_{NB} + \mathrm{Res}_{I,J}}{a_{I,J}}$$

$$u^{k+1} = \alpha_u(u^* + u') + (1-\alpha_u)u^k$$

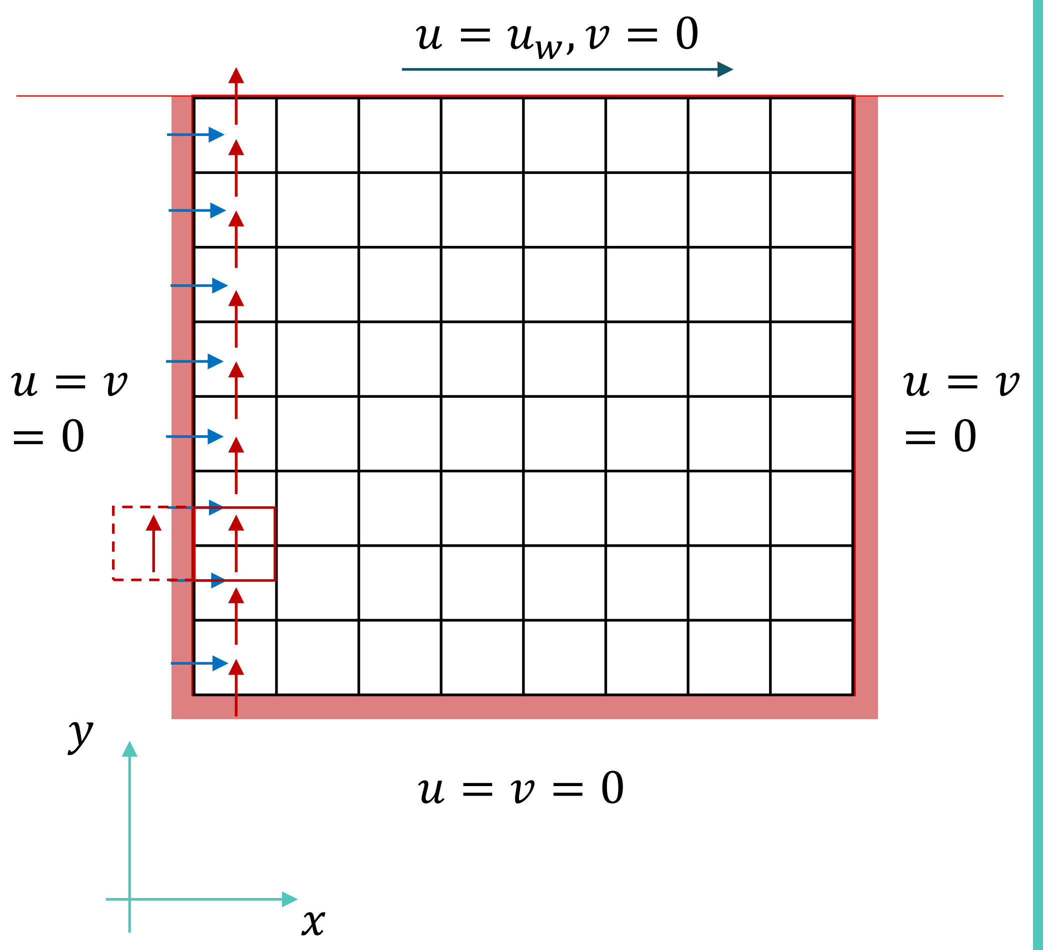
$$v^{k+1} = \alpha_v(v^* + v') + (1-\alpha_v)v^k$$



معادله $N-S$

مش جابجا شده

شرایط مرزی سرعت



دیواره شرقی و غربی:

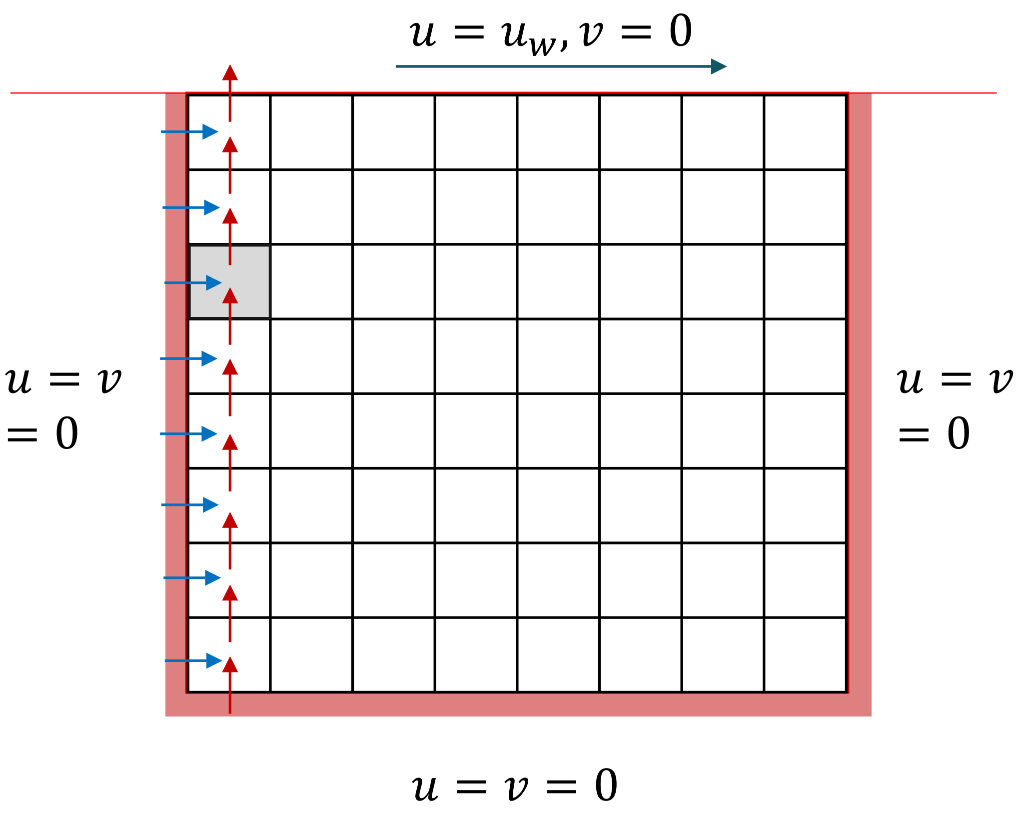
$$u = 0$$

$$\frac{v_{0,j} + v_{1,j}}{2} = 0$$

$$\frac{v_{N,j} + v_{N+1,j}}{2} = 0$$

مش جابجا شده

شرایط مرزی فشار



دیواره غربی:

$$a_{I,J}^p p'_{I,J} + \sum a_{NB}^p p'_{NB} = \mathrm{Res}_{I,J}$$

$$\frac{u_{i+1,J} - u_{i,J}}{\Delta x} + \frac{v_{I,j+1} - v_{I,j}}{\Delta y} = 0$$

$$\frac{(u_{i+1,J}^* + u'_{i+1,J}) - 0}{\Delta x} + \frac{(v_{I,j+1}^* + v'_{I,j+1}) - (v_{I,j}^* + v'_{I,j})}{\Delta y} = 0$$

$$\begin{cases} u'_{i,J} = \dfrac{-p'_{I,J} + p'_{I-1,J}}{a_{i,J}^u \Delta x} \\ v'_{I,j} = \dfrac{-p'_{I,J} + p'_{I,J-1}}{a_{I,j}^v \Delta y} \end{cases}$$

$$a_{I,J}^p p'_{I,J} + a_{I+1,J}^p p'_{I+1,J} + a_{I,J+1}^p p'_{I,J+1} + a_{I,J-1}^p p'_{I,J-1} = \mathrm{Res}_{I,J}$$

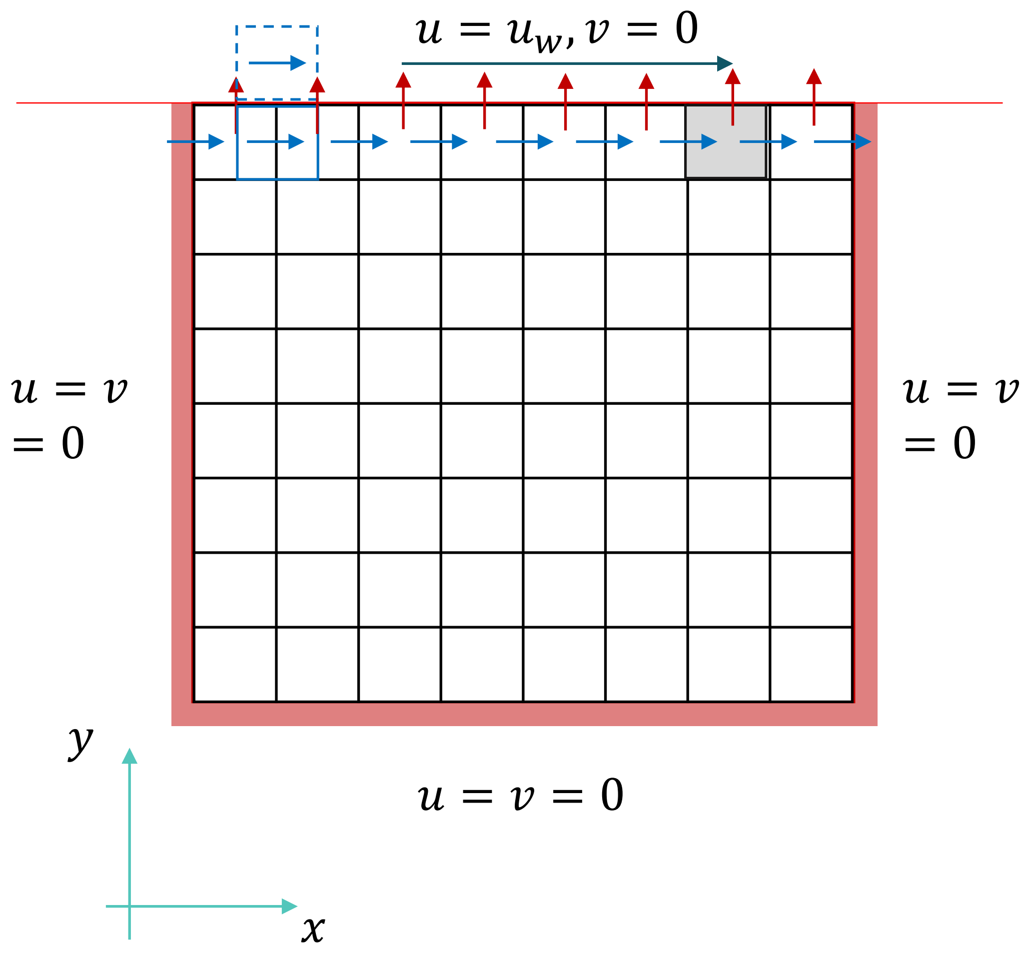
دیواره شرقی و غربی:

$$\boxed{a_{0,J}^p = a_{N+1,J}^p = 0}$$

معادله $N-S$

مش جابجا شده

شرایط مرزی



دیواره شمالی و جنوبی:

$$v = 0$$

$$\frac{u_{i,0} + u_{i,1}}{2} = 0$$

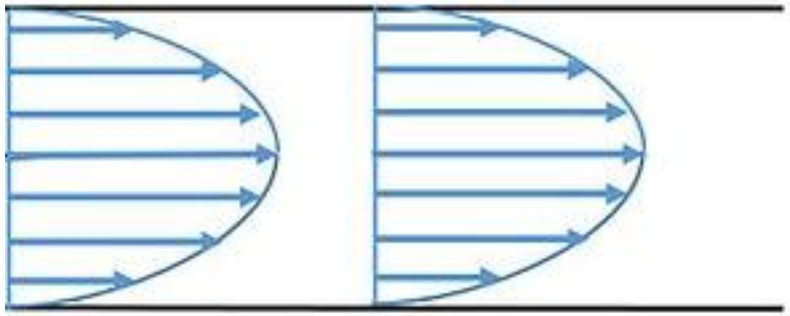
$$\frac{u_{i,M} + v_{i,M+1}}{2} = u_w$$

$$\boxed{a^p_{I,0} = a^p_{I,M+1} = 0}$$

معادله $N-S$

مش جابجا شده

شرایط مرزی



$$u_{i,J} = u_{\text{inlet}}\Big|_{x=0,y=y_J}$$

$$\frac{v_{0,j}+v_{1,j}}{2} = 0$$

$$a_{0,J}^{p} = 0$$

$$p'_{N,J} = 0$$

توسعه یافته:
$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x} = 0$$

توسعه نیافته: بدست آوردن سرعت از دو سلول قبلی

حجم محدود روی شبکه یکسان (Collocated grid)

$$\frac{\partial u\phi}{\partial x}+\frac{\partial v\phi}{\partial y}=\alpha\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\right)+S$$

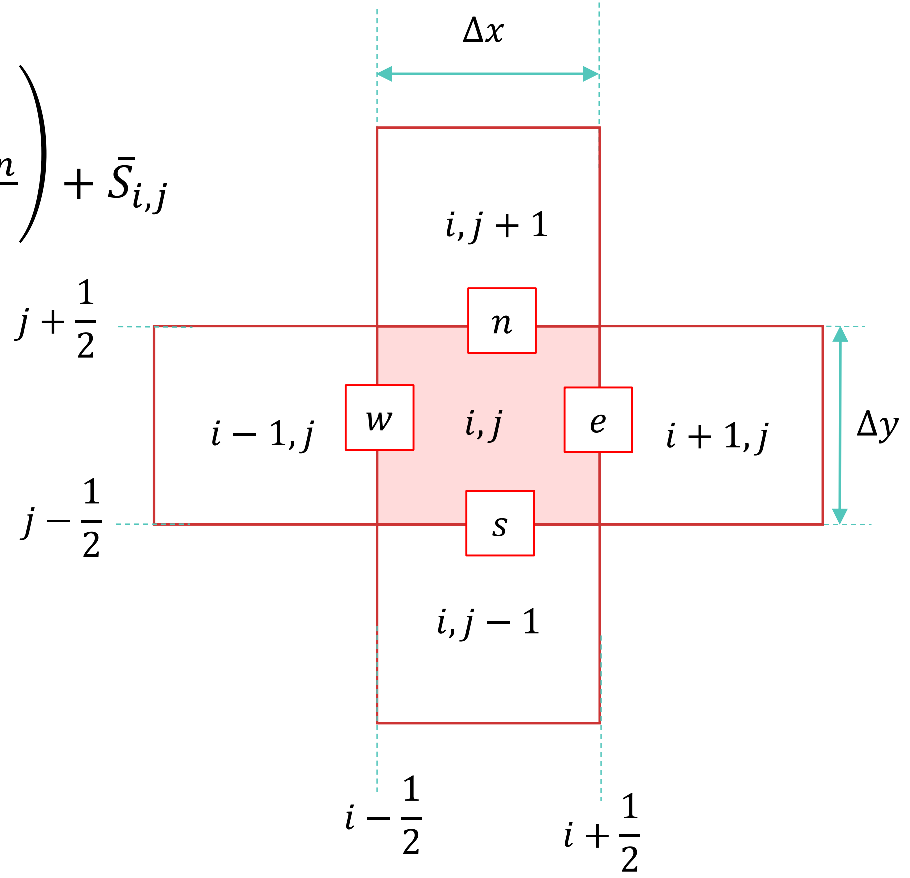
$$\frac{u_e\phi_e-u_w\phi_w}{\Delta x}+\frac{v_n\phi_n-v_s\phi_s}{\Delta y}=\alpha\left(\frac{\left.\frac{\partial\phi}{\partial x}\right|_e-\left.\frac{\partial\phi}{\partial x}\right|_w}{\Delta x}+\frac{\left.\frac{\partial\phi}{\partial y}\right|_n-\left.\frac{\partial\phi}{\partial y}\right|_n}{\Delta y}\right)+\bar{S}_{i,j}$$

$$u_e=\frac{\bar{u}_{i+1,j}+\bar{u}_{i,j}}{2}+O(\Delta x^2)$$

$$\phi_e=\frac{\bar{\phi}_{i+1,j}+\bar{\phi}_{i,j}}{2}+O(\Delta x^2)$$

$$\phi_w=\frac{\bar{\phi}_{i,j}+\bar{\phi}_{i-1,j}}{2}+O(\Delta x^2)$$

$$\left.\frac{\partial\phi}{\partial x}\right|_e=\frac{\bar{\phi}_{i+1,j}-\bar{\phi}_{i,j}}{\Delta x}+O(\Delta x^2)$$



معادله $N-S$

گسسته‌سازی مرکزی

$$a_P\bar{\phi}_P + \sum a_{NB}\bar{\phi}_{NB} = \bar{S}_P$$

$$a_P\bar{\phi}_P + a_E\bar{\phi}_E + a_W\bar{\phi}_W + a_N\bar{\phi}_N + a_S\bar{\phi}_S = \bar{S}_P$$

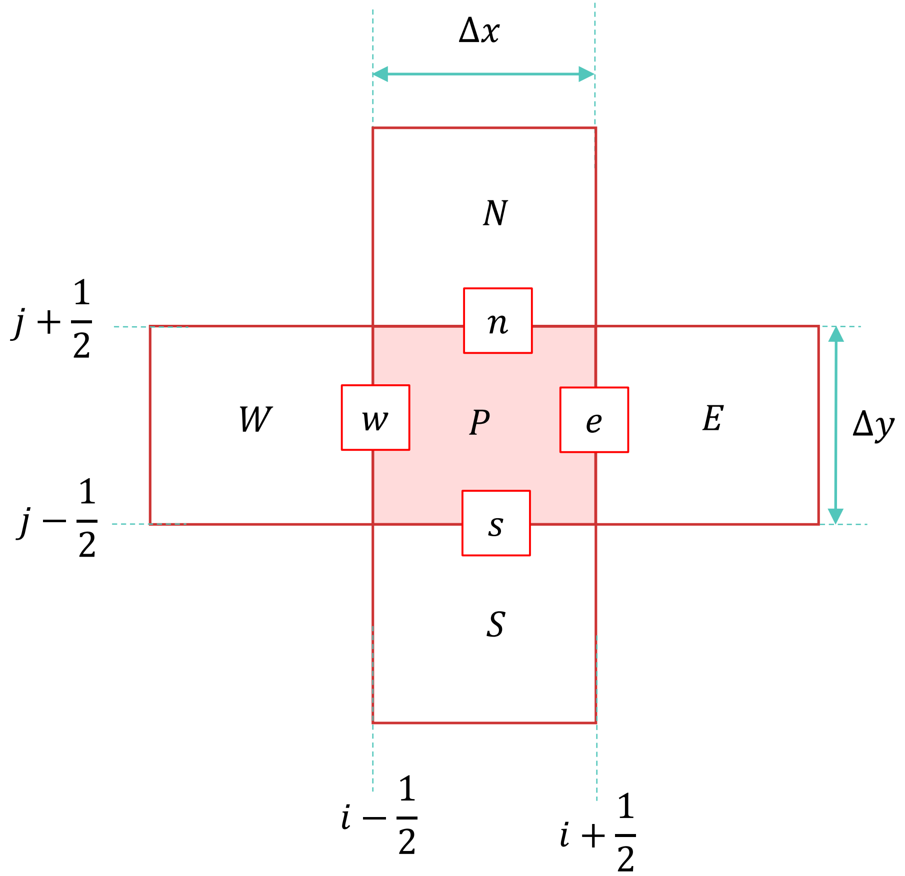
$$a_E = \frac{u_e}{2\Delta x} - \frac{1}{Re\Delta x^2}$$

$$a_W = \frac{-u_w}{2\Delta x} - \frac{1}{Re\Delta x^2}$$

$$a_N = \frac{v_n}{2\Delta y} - \frac{1}{Re\Delta y^2}$$

$$a_S = \frac{-v_s}{2\Delta y} - \frac{1}{Re\Delta y^2}$$

$$a_P = \frac{u_e - u_w}{2\Delta x} + \frac{v_n - v_s}{2\Delta y} + \frac{2}{Re\Delta x^2} + \frac{2}{Re\Delta y^2}$$



معادله $N-S$

$$\frac{\partial u\phi}{\partial x}+\frac{\partial v\phi}{\partial y}=\alpha\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\right)+S$$

$$a_P\bar{\phi}_P+\sum a_{NB}\bar{\phi}_{NB}=\bar{S}_P$$

$x-\text{momentum:}$

$$\frac{\partial(uu)}{\partial x}+\frac{\partial(vu)}{\partial y}=\frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\partial p}{\partial x}$$

$$\phi=u \qquad \alpha=\frac{1}{Re} \qquad S=-\frac{\partial p}{\partial x}$$

$$a_P\bar{u}_P+\sum a_{NB}\bar{u}_{NB}=\bar{S}_P^u$$

$y-\text{momentum:}$

$$\frac{\partial(uv)}{\partial x}+\frac{\partial(vv)}{\partial y}=\frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\partial p}{\partial y}$$

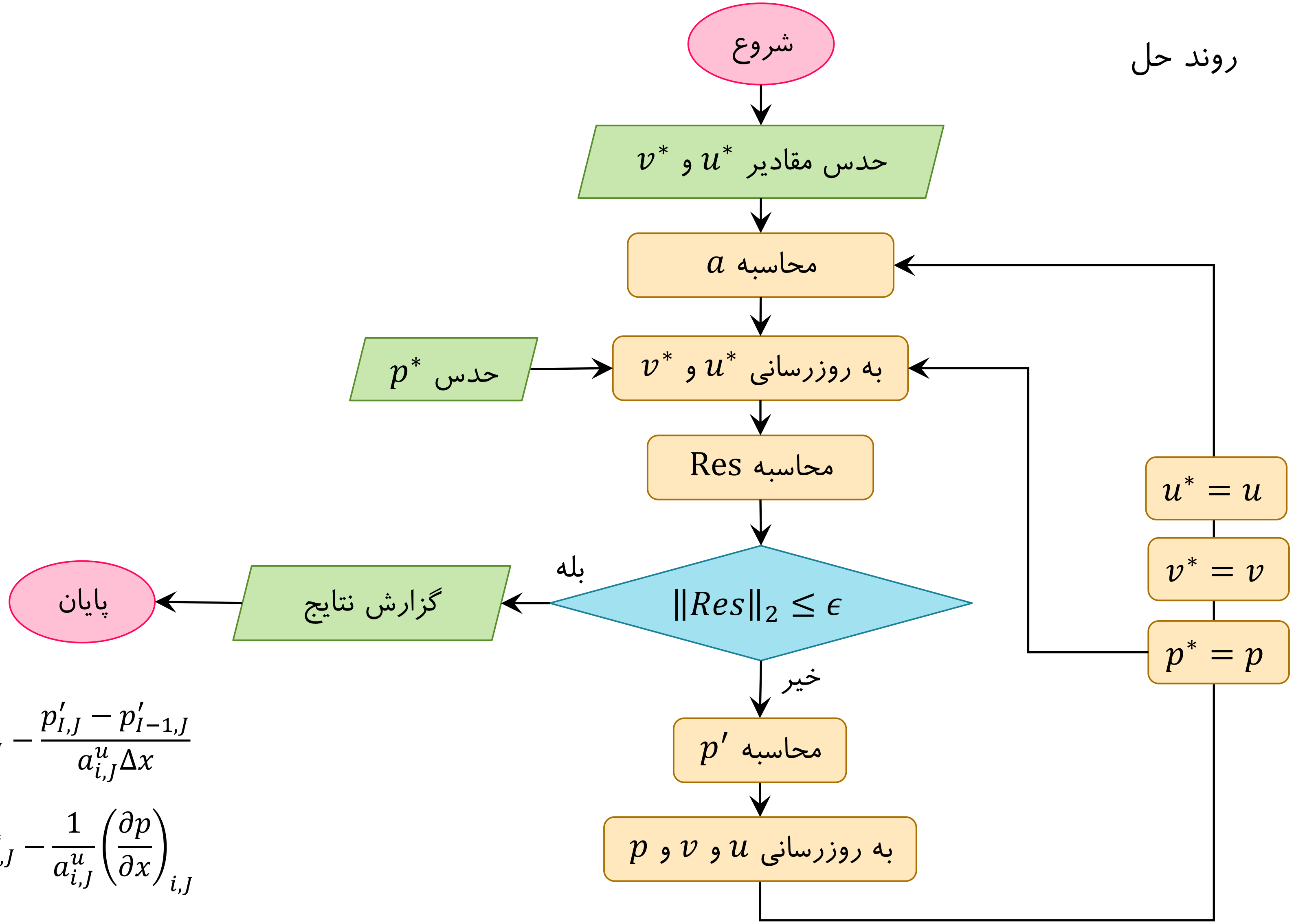
$$a_P\bar{v}_P+\sum a_{NB}\bar{v}_{NB}=\bar{S}_P^v$$

$$\phi=v \qquad \alpha=\frac{1}{Re} \qquad S=-\frac{\partial p}{\partial y}$$

الگوریتم SIMPLE

روند حل

$$\begin{cases} a_{i,j}u^*_{i,j} + \sum a_{nb}u^*_{nb} = \dfrac{-p^*_{i+1,j} + p^*_{i-1,j}}{2\Delta x} \\ a_{i,j}v^*_{i,j} + \sum a_{nb}v^*_{nb} = \dfrac{-p^*_{i,j+1} + p^*_{i,j-1}}{2\Delta y} \end{cases}$$



Staggered grid: $u_{i,J} = u^*_{i,J} + u'_{i,J} = u^*_{i,J} - \dfrac{p'_{I,J} - p'_{I-1,J}}{a^u_{i,J}\Delta x}$

$$= u^*_{i,J} - \frac{1}{a^u_{i,J}}\left(\frac{\partial p}{\partial x}\right)_{i,J}$$

معادله $N-S$

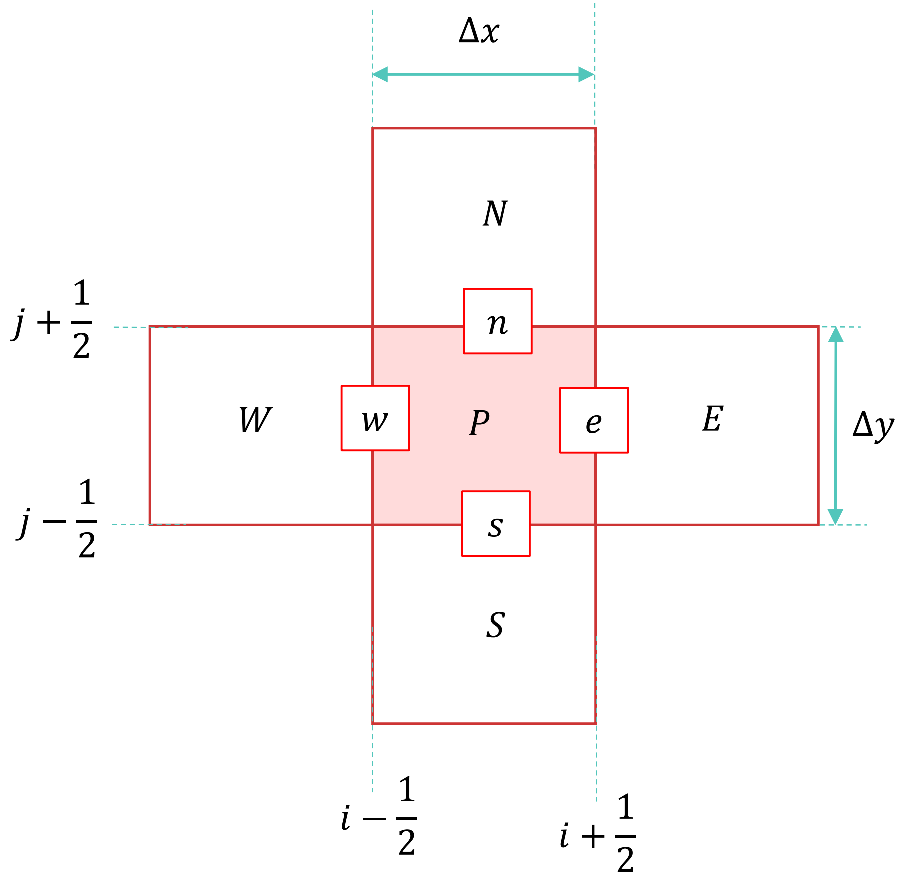
اصلاح فشار

$$u_e = u_e^* - \frac{p'_{i+1,j} - p'_{i,j}}{a_e \Delta x}$$

$$u_w = u_w^* - \frac{p'_{i,j} - p'_{i-1,j}}{a_w \Delta x}$$

$$v_n = v_n^* - \frac{p'_{i,j+1} - p'_{i,j}}{a_n \Delta y}$$

$$v_s = v_s^* - \frac{p'_{i,j} - p'_{i,j-1}}{a_s \Delta y}$$



$$a_e = \frac{1}{2}\left(a_P\Big|_{i,j} + a_P\Big|_{i+1,j}\right)$$

یا

$$\frac{1}{a_e} = \frac{1}{2}\left(\frac{1}{a_P|_{i,j}} + \frac{1}{a_P|_{i+1,j}}\right)$$

معادله $N-S$

حجم محدود روی شبکه یکسان

اصلاح فشار

معادله پیوستگی
$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \frac{u_e-u_w}{\Delta x}+\frac{v_n-v_s}{\Delta y}=0$$

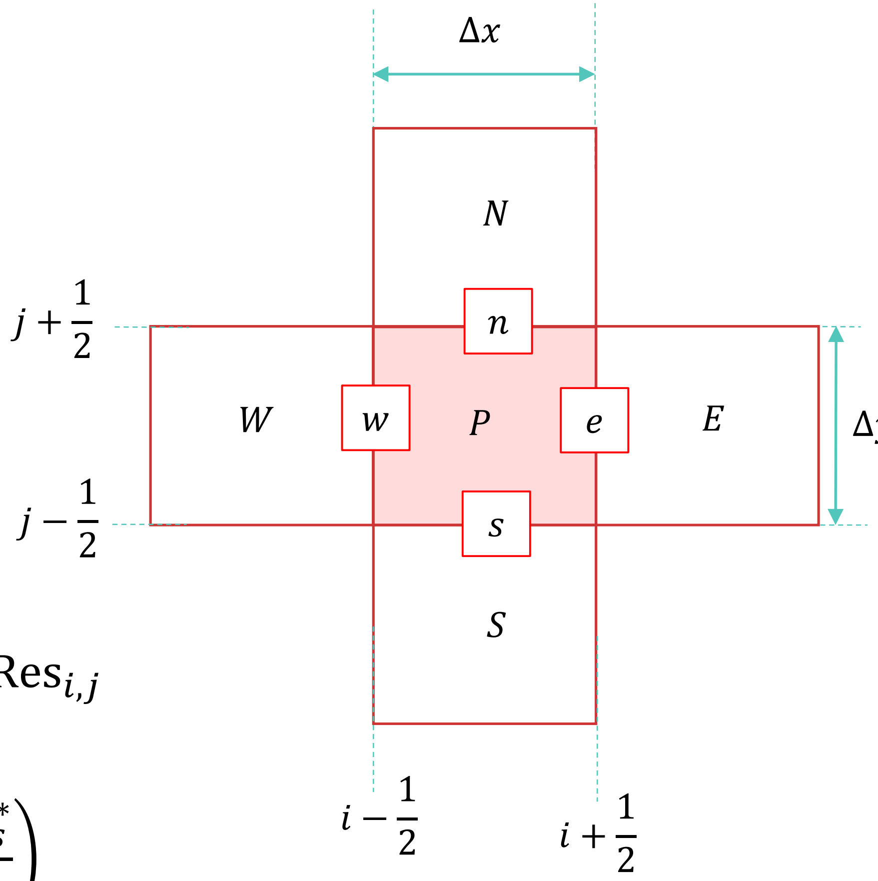
$$\frac{u_e^*}{\Delta x}-\frac{p'_{i+1,j}-p'_{i,j}}{a_e\Delta x^2}-\frac{u_w^*}{\Delta x}+\frac{p'_{i,j}-p'_{i-1,j}}{a_w\Delta x^2}$$

$$+\frac{v_n^*}{\Delta y}-\frac{p'_{i,j+1}-p'_{i,j}}{a_n\Delta y^2}-\frac{v_s^*}{\Delta y}+\frac{p'_{i,j}-p'_{i,j-1}}{a_s\Delta y^2}=0$$

$$b_{i,j}\,p'_{i,j}+b_{i+1,j}\,p'_{i+1,j}+b_{i,j+1}\,p'_{i,j+1}+b_{i-1,j}\,p'_{i-1,j}+b_{i,j-1}\,p'_{i,j-1}=\mathrm{Res}_{i,j}$$

$$b_P\,p'_P+\sum b_{NB}p'_{NB}=\mathrm{Res}_P \qquad \mathrm{Res}_P=-\left(\frac{u_e^*-u_w^*}{\Delta x}+\frac{v_n^*-v_s^*}{\Delta y}\right)$$

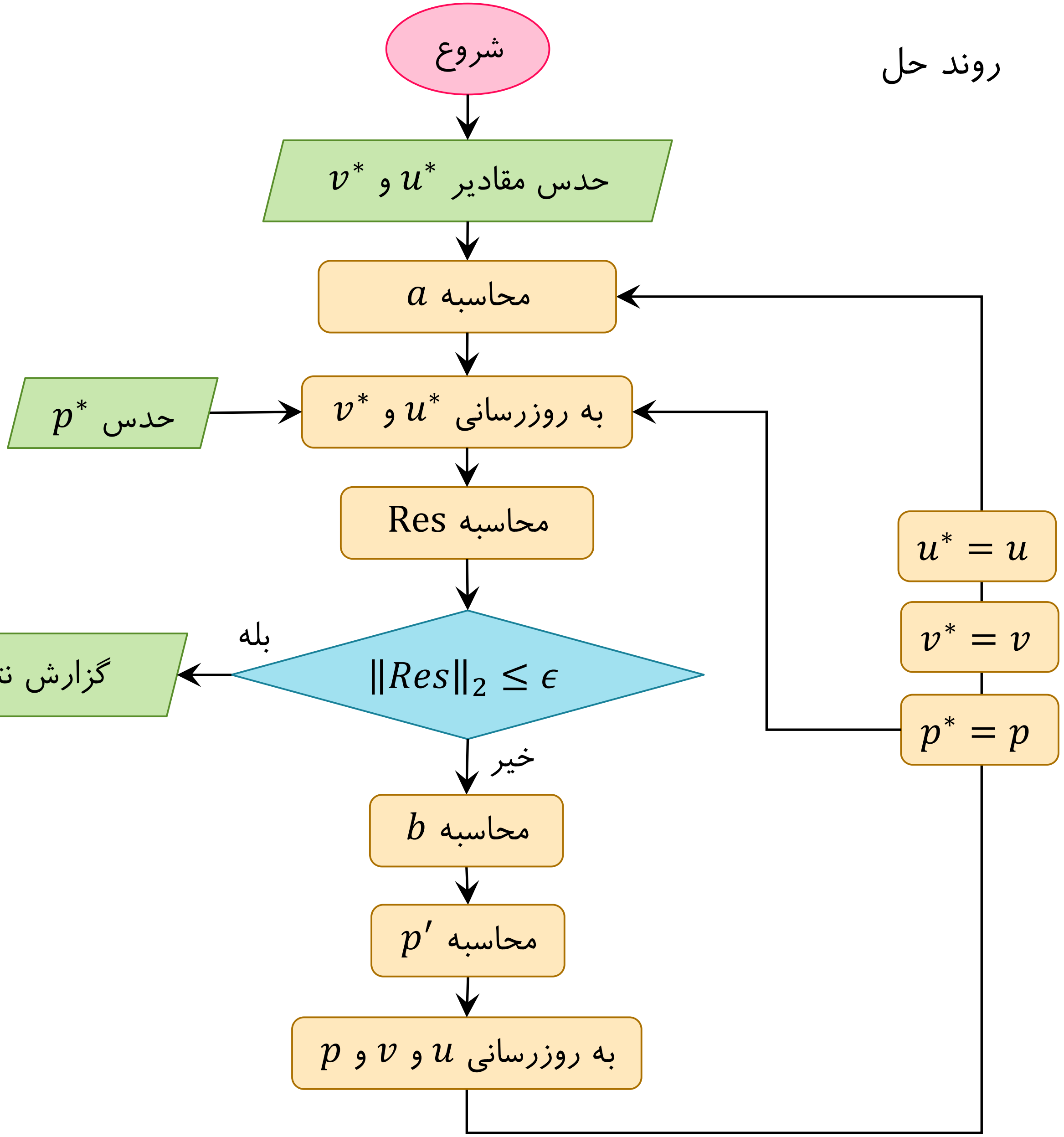
$$b_P=\sum b_{NB}$$



معادله $N-S$

الگوریتم SIMPLE

روند حل



$$\begin{cases} a_{i,j}u^*_{i,j} + \sum a_{nb}u^*_{nb} = \dfrac{-p^*_{i+1,j} + p^*_{i-1,j}}{2\Delta x} \\ a_{i,j}v^*_{i,j} + \sum a_{nb}v^*_{nb} = \dfrac{-p^*_{i,j+1} + p^*_{i,j-1}}{2\Delta y} \end{cases}$$

$$\mathrm{Res}_{i,j} = -\left(\frac{u^*_e - u^*_w}{\Delta x} + \frac{v^*_n - v^*_s}{\Delta y}\right)$$

$$u_{i,j} = u^*_{i,j} + u'_{i,j} \qquad = u^*_{i,j} - \frac{p'_{i+1,j} - p'_{i-1,j}}{2a_P|_{i,j}\Delta x}$$

$$v_{i,j} = v^*_{i,j} + v'_{i,j} \qquad = v^*_{i,j} - \frac{p'_{i,j+1} - p'_{i,j-1}}{2a_P|_{i,j}\Delta y}$$

معادله $N-S$

$$\mathrm{Res}_P = -\left(\frac{u_e^* - u_w^*}{\Delta x} + \frac{v_n^* - v_s^*}{\Delta y}\right)$$

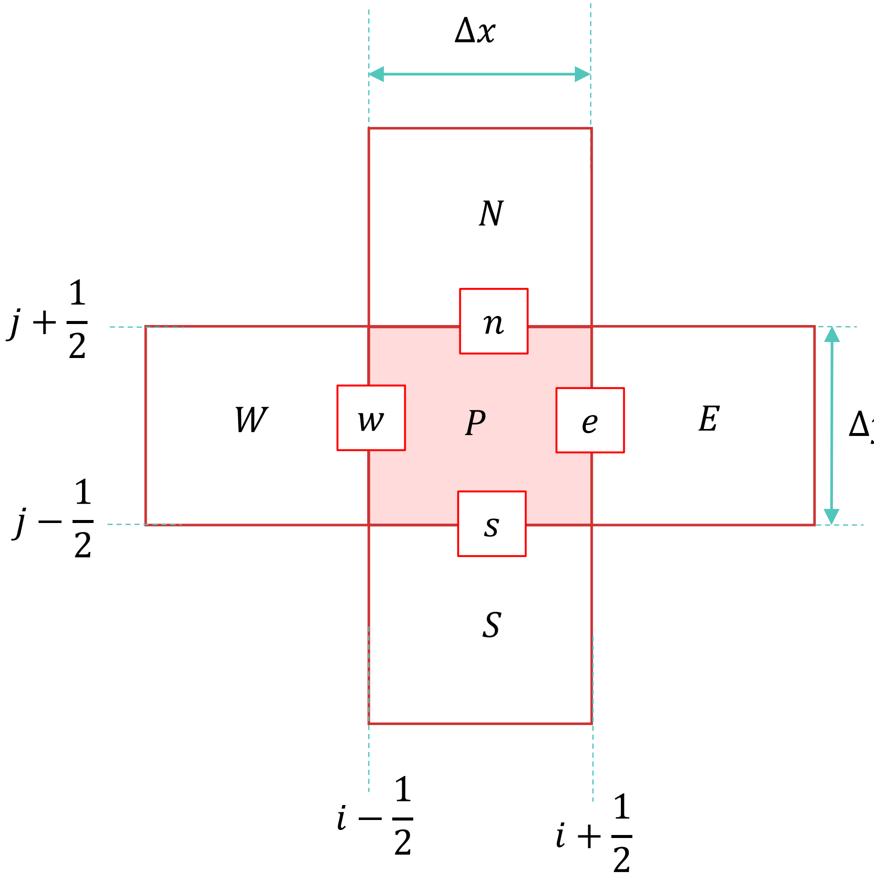
$$u_e^* = \frac{1}{2}(\bar{u}_P + \bar{u}_E)$$

$$u_w^* = \frac{1}{2}(\bar{u}_P + \bar{u}_W)$$

$$v_n^* = \frac{1}{2}(\bar{v}_P + \bar{v}_N)$$

$$v_s^* = \frac{1}{2}(\bar{v}_P + \bar{v}_S)$$

$$\mathrm{Res}_P = -\left(\frac{\bar{u}_E - \bar{u}_W}{2\Delta x} + \frac{\bar{v}_N - \bar{v}_S}{2\Delta y}\right)$$



معادله $N-S$

حجم محدود روی شبکه یکسان

درون‌یابی رای-چو (Rhie - Chow)

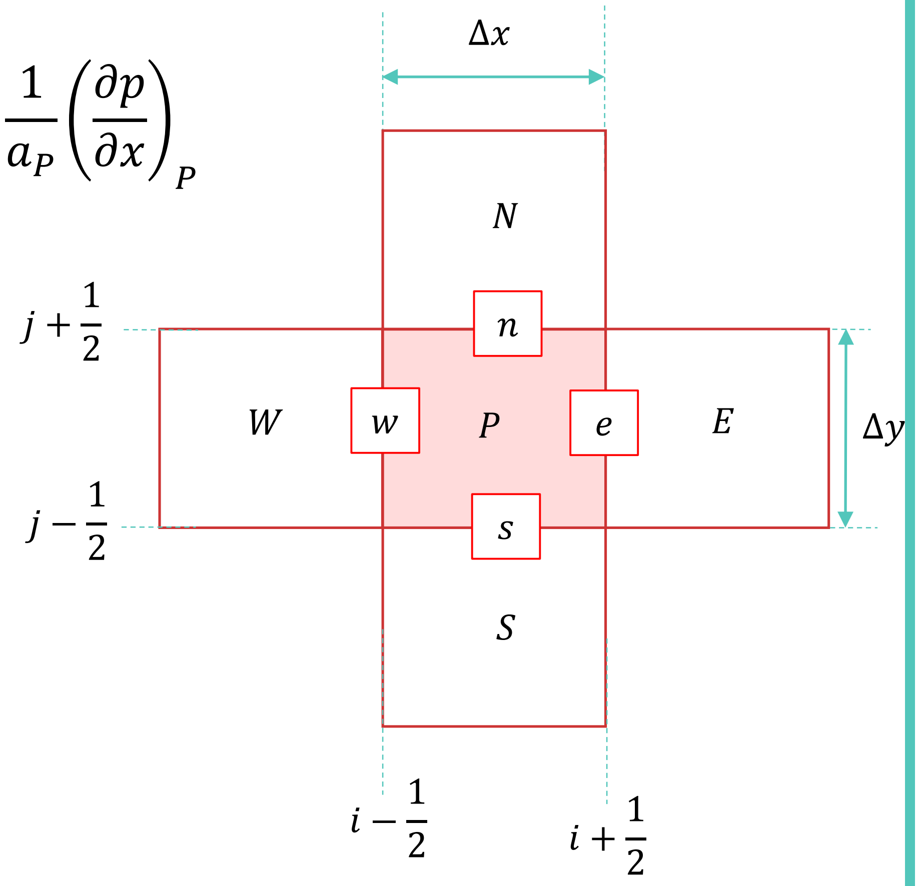
$$a_P\bar{\phi}_P + \sum a_{NB}\bar{\phi}_{NB} = \bar{S}_P$$

$$\bar{u}_P + \frac{1}{a_P}\sum a_{NB}\bar{u}_{NB} = -\frac{1}{a_P}\left(\frac{\partial p}{\partial x}\right)_P \qquad u'_P + \underbrace{\cancel{\frac{1}{a_P}\sum a_{NB}u'_{NB}}}_{0} = -\frac{1}{a_P}\left(\frac{\partial p}{\partial x}\right)_P$$

$$\left(\frac{\partial p}{\partial x}\right)_P = \frac{p_E - p_W}{2\Delta x}$$

$$\bar{u}_P + \frac{1}{a_P}\left(\frac{\partial p}{\partial x}\right)_P = \frac{-1}{a_p}\sum a_{NB}\bar{u}_{NB}$$

$$u_e + \frac{1}{a_e}\left(\frac{\partial p}{\partial x}\right)_e = ?$$



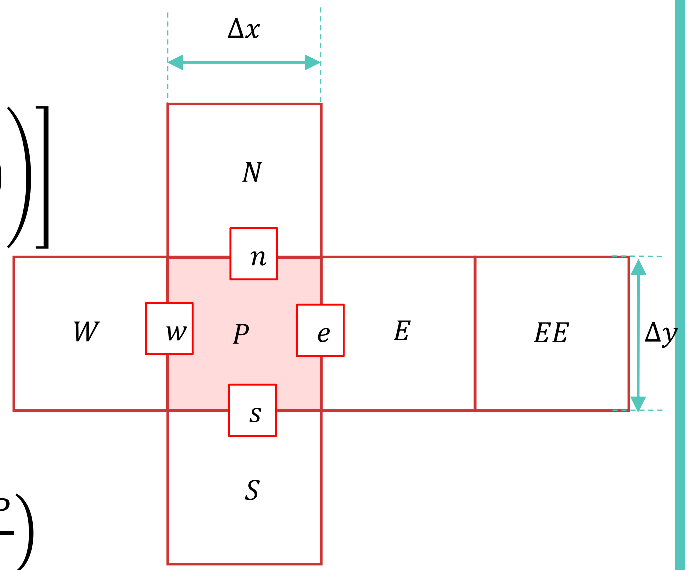
معادله $N - S$

حجم محدود روی شبکه یکسان

درون‌یابی رای-چو (Rhie - Chow)

$$u_e + \frac{1}{a_e}\left(\frac{\partial p}{\partial x}\right)_e = \frac{1}{2}\left[\left(u_P + \frac{1}{a_P|_{i,j}}\left(\frac{\partial p}{\partial x}\right)_P\right) + \left(u_E + \frac{1}{a_P|_{i+1,j}}\left(\frac{\partial p}{\partial x}\right)_E\right)\right]$$

$$u_e + \frac{1}{a_e}\left(\frac{p_E - p_P}{\Delta x}\right) = \frac{1}{2}\left[\left(u_P + \frac{1}{a_P|_{i,j}}\left(\frac{p_E - p_W}{2\Delta x}\right)\right) + \left(u_E + \frac{1}{a_P|_{i+1,j}}\left(\frac{p_{EE} - p_P}{2\Delta x}\right)\right)\right]$$



$$u_e = \frac{1}{2}\left[\left(u_P + \frac{1}{a_P|_{i,j}}\left(\frac{p_E - p_W}{2\Delta x}\right)\right) + \left(u_E + \frac{1}{a_P|_{i+1,j}}\left(\frac{p_{EE} - p_P}{2\Delta x}\right)\right)\right] - \frac{1}{a_e}\left(\frac{p_E - p_P}{\Delta x}\right)$$

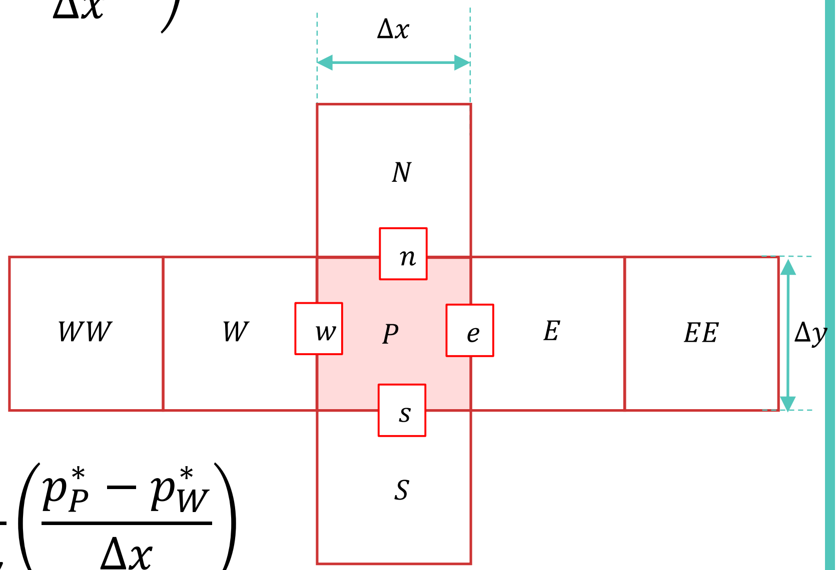
$$\frac{1}{a_e} = \frac{1}{2}\left(\frac{1}{a_P|_{i,j}} + \frac{1}{a_P|_{i+1,j}}\right)$$

معادله $N-S$

درون‌یابی رای-چو (Rhie - Chow)

$$u_e^* = \frac{1}{2}\left[\left(u_P^* + \frac{1}{a_P|_{i,j}}\left(\frac{p_E^* - p_W^*}{2\Delta x}\right)\right) + \left(u_E^* + \frac{1}{a_P|_{i+1,j}}\left(\frac{p_{EE}^* - p_P^*}{2\Delta x}\right)\right)\right] - \frac{1}{a_e}\left(\frac{p_E^* - p_P^*}{\Delta x}\right)$$

$$\frac{1}{a_e} = \frac{1}{2}\left(\frac{1}{a_P|_{i,j}} + \frac{1}{a_P|_{i+1,j}}\right)$$



$$u_w^* = \frac{1}{2}\left[\left(u_P^* + \frac{1}{a_P|_{i,j}}\left(\frac{p_E^* - p_W^*}{2\Delta x}\right)\right) + \left(u_W^* + \frac{1}{a_P|_{i-1,j}}\left(\frac{p_P^* - p_{WW}^*}{2\Delta x}\right)\right)\right] - \frac{1}{a_w}\left(\frac{p_P^* - p_W^*}{\Delta x}\right)$$

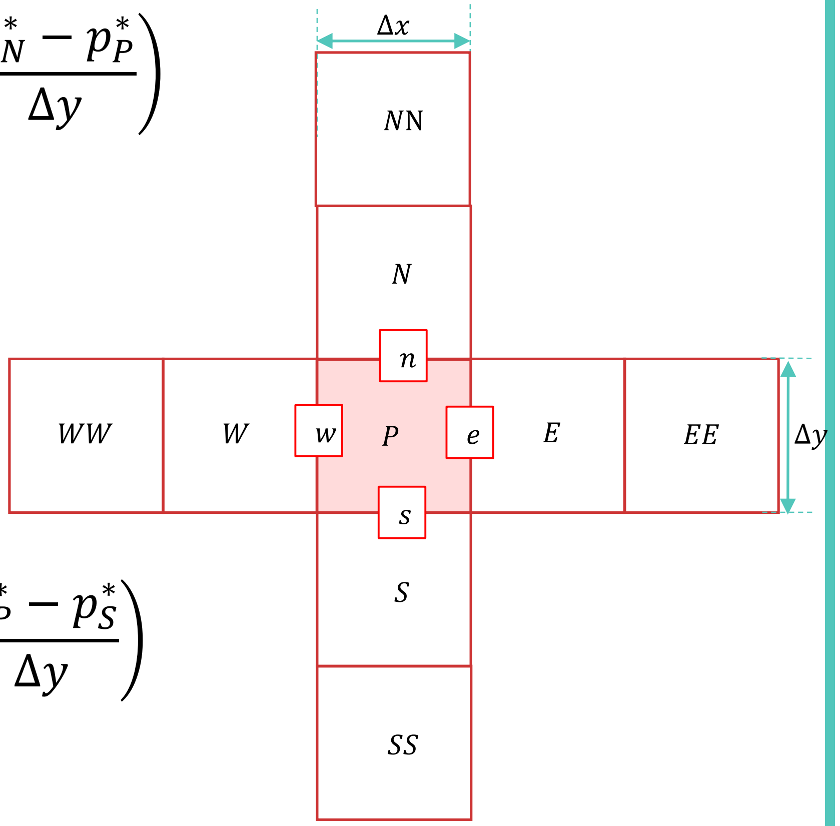
$$\frac{1}{a_w} = \frac{1}{2}\left(\frac{1}{a_P|_{i,j}} + \frac{1}{a_P|_{i-1,j}}\right)$$

معادله $N-S$

درون‌یابی رای-چو (Rhie - Chow)

$$v_n^* = \frac{1}{2}\left[\left(v_P^* + \frac{1}{a_P|_{i,j}}\left(\frac{p_N^* - p_S^*}{2\Delta y}\right)\right) + \left(v_N^* + \frac{1}{a_P|_{i,j+1}}\left(\frac{p_{NN}^* - p_P^*}{2\Delta y}\right)\right)\right] - \frac{1}{a_n}\left(\frac{p_N^* - p_P^*}{\Delta y}\right)$$

$$\frac{1}{a_n} = \frac{1}{2}\left(\frac{1}{a_P|_{i,j}} + \frac{1}{a_P|_{i,j+1}}\right)$$



$$v_s^* = \frac{1}{2}\left[\left(v_P^* + \frac{1}{a_P|_{i,j}}\left(\frac{p_N^* - p_S^*}{2\Delta y}\right)\right) + \left(v_S^* + \frac{1}{a_P|_{i,j-1}}\left(\frac{p_P^* - p_{SS}^*}{2\Delta y}\right)\right)\right] - \frac{1}{a_s}\left(\frac{p_P^* - p_S^*}{\Delta y}\right)$$

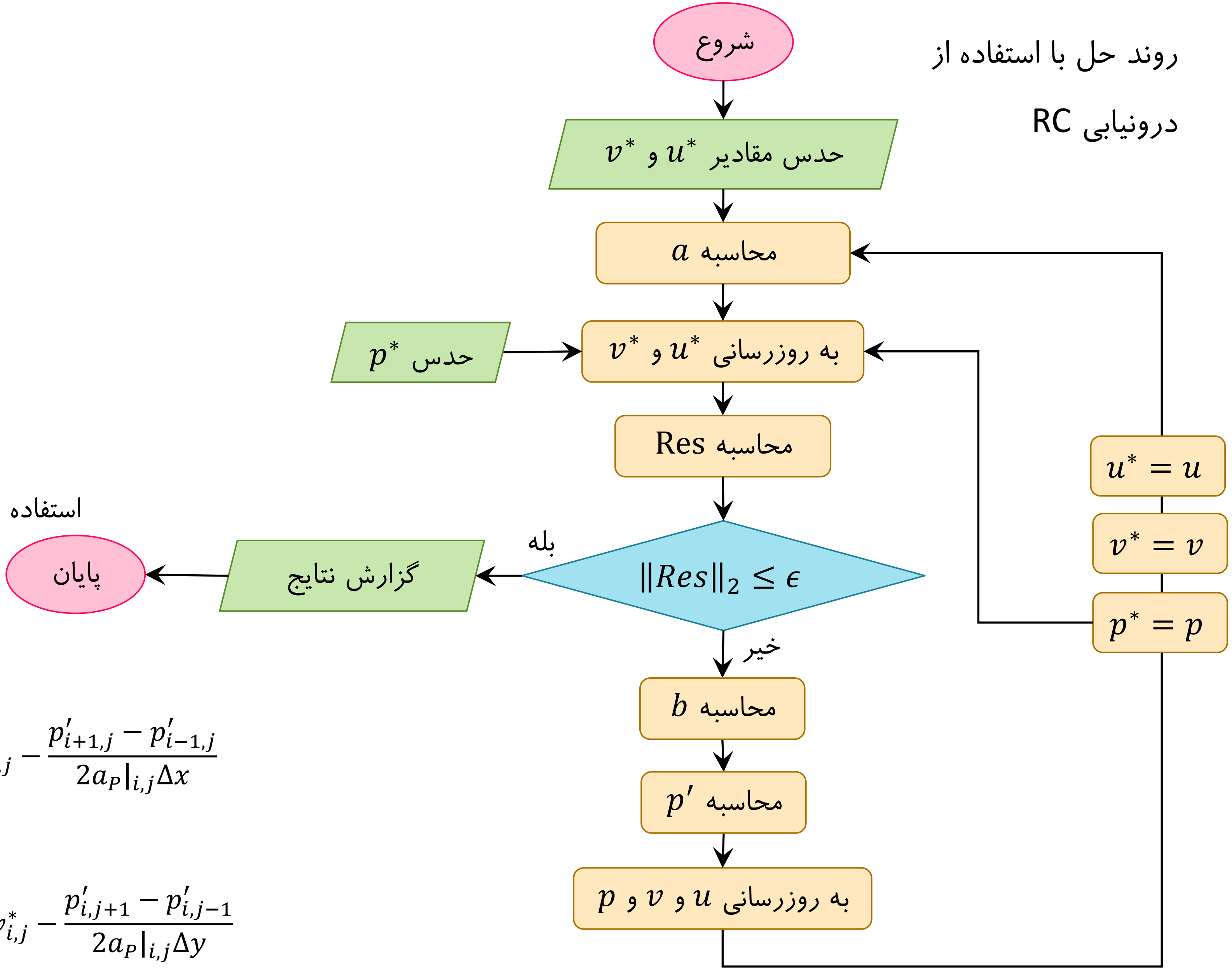
$$\frac{1}{a_s} = \frac{1}{2}\left(\frac{1}{a_P|_{i,j}} + \frac{1}{a_P|_{i,j-1}}\right)$$

معادله $N-S$

الگوریتم SIMPLE

روند حل با استفاده از

درونیابی RC



$$\begin{cases} a_{i,j}u^*_{i,j} + \sum a_{nb}u^*_{nb} = \dfrac{-p^*_{i+1,j} + p^*_{i-1,j}}{2\Delta x} \\ a_{i,j}v^*_{i,j} + \sum a_{nb}v^*_{nb} = \dfrac{-p^*_{i,j+1} + p^*_{i,j-1}}{2\Delta y} \end{cases}$$

استفاده از درونیابی RC

$$\mathrm{Res}_{i,j} = -\left(\frac{u^*_e - u^*_w}{\Delta x} + \frac{v^*_n - v^*_s}{\Delta y}\right)$$

$$u_{i,j} = u^*_{i,j} + u'_{i,j} \qquad = u^*_{i,j} - \frac{p'_{i+1,j} - p'_{i-1,j}}{2a_P|_{i,j}\Delta x}$$

$$v_{i,j} = v^*_{i,j} + v'_{i,j} \qquad = v^*_{i,j} - \frac{p'_{i,j+1} - p'_{i,j-1}}{2a_P|_{i,j}\Delta y}$$

معادله $N-S$

درون‌یابی رای-چو (Rhie - Chow)

$$a_P = \frac{u_e - u_w}{2\Delta x} + \frac{v_n - v_s}{2\Delta y} + \frac{2}{Re\Delta x^2} + \frac{2}{Re\Delta y^2}$$

فرض: a_P ثابت است.

$$u_e = \frac{1}{2}(\bar{u}_P + \bar{u}_E) + \frac{1}{4a_p\Delta x}(\bar{p}_{EE} - 3\bar{p}_E + 3\bar{p}_P - \bar{p}_W)$$

$$u_w = \frac{1}{2}(\bar{u}_P + \bar{u}_W) + \frac{1}{4a_p\Delta x}(\bar{p}_E - 3\bar{p}_P + 3\bar{p}_W - \bar{p}_{WW})$$

$$v_n = \frac{1}{2}(\bar{v}_P + \bar{v}_N) + \frac{1}{4a_p\Delta y}(\bar{p}_{NN} - 3\bar{p}_N + 3\bar{p}_P - \bar{p}_S)$$

$$v_s = \frac{1}{2}(\bar{v}_P + \bar{v}_S) + \frac{1}{4a_p\Delta y}(\bar{p}_N - 3\bar{p}_P + 3\bar{p}_S - \bar{p}_{SS})$$

معادله $N-S$

حجم محدود روی شبکه یکسان

درون‌یابی رای-چو (Rhie - Chow)

فرض: a_P ثابت است.

$$\frac{\partial u}{\partial x}=\frac{1}{4a_p}\frac{\partial^4 p}{\partial x^4}\Delta x^2$$

$$\frac{u_e-u_w}{\Delta x}=\frac{1}{2\Delta x}(\bar{u}_E-\bar{u}_W)+\frac{1}{4a_p\Delta x^2}(\bar{p}_{EE}-4\bar{p}_E+6\bar{p}_P-4\bar{p}_W+\bar{p}_{WW})$$

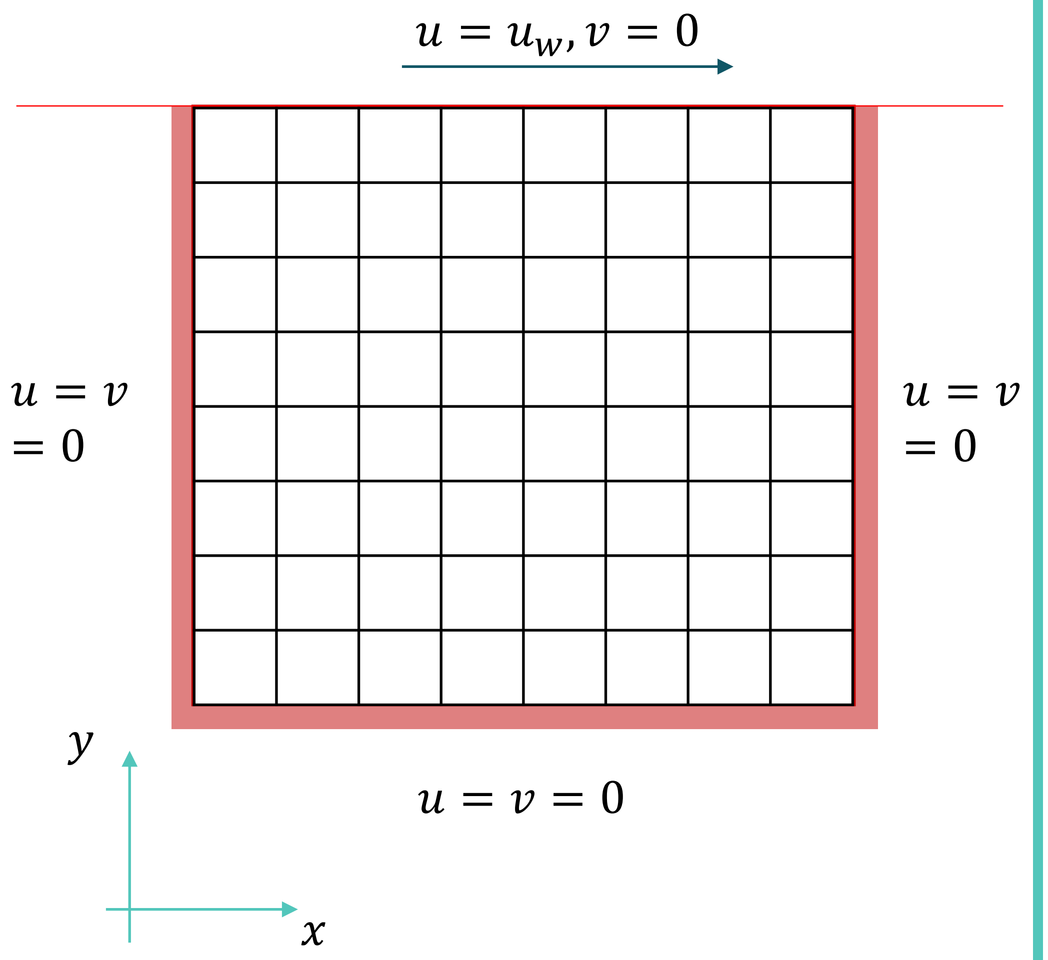
$$\frac{\partial v}{\partial y}=\frac{1}{4a_p}\frac{\partial^4 p}{\partial y^4}\Delta y^2$$

$$\frac{v_n-v_s}{\Delta y}=\frac{1}{2\Delta y}(\bar{v}_N-\bar{v}_S)+\frac{1}{4a_p\Delta y^2}(\bar{p}_{NN}-4\bar{p}_N+6\bar{p}_P-4\bar{p}_S+\bar{p}_{SS})$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=C\left(\frac{\partial^4 p}{\partial x^4}\Delta x^2+\frac{\partial^4 p}{\partial y^4}\Delta y^2\right)$$

Artificial Dissipation (AD)

شرایط مرزی



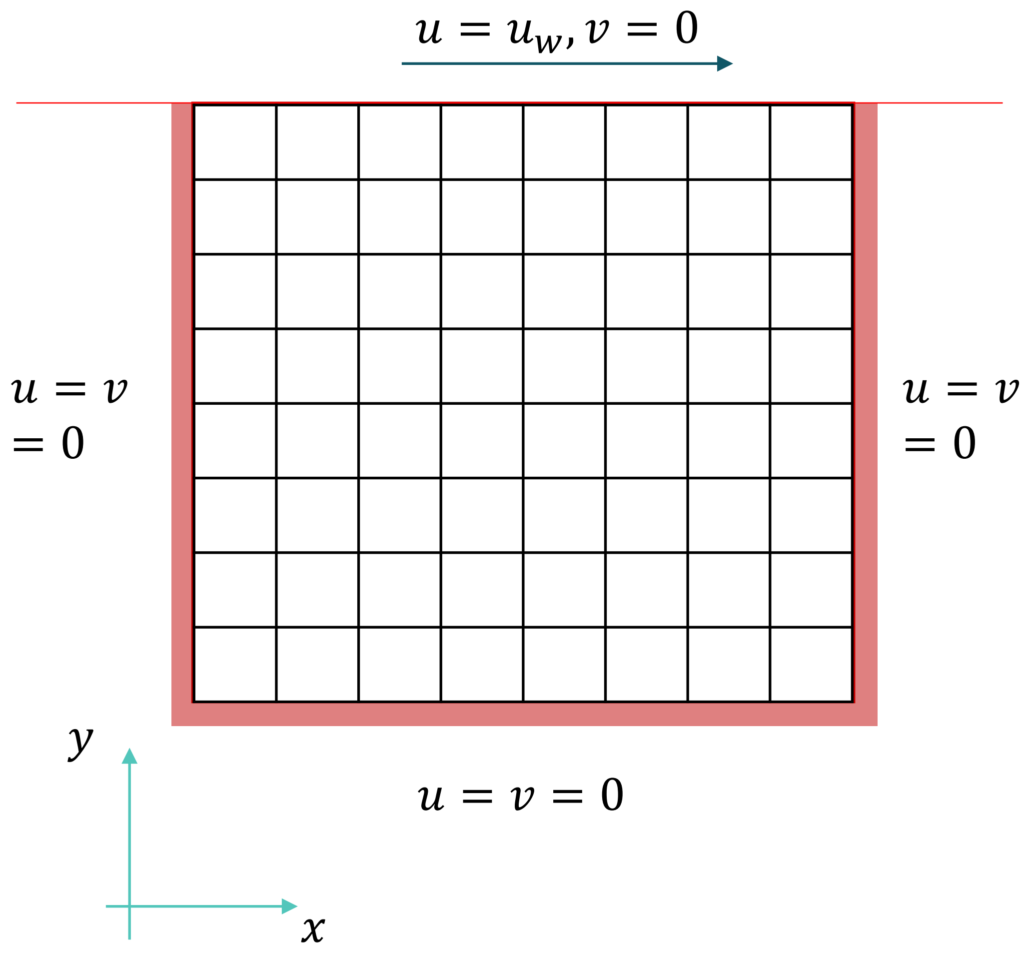
دیواره شمالی و جنوبی:

$y-$momentum:

$$u\cancelto{0}{\frac{\partial v}{\partial x}}+\cancelto{0}{v}\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\left(\cancelto{0}{\frac{\partial^2 v}{\partial x^2}}+\frac{\partial^2 v}{\partial y^2}\right)$$

$$\frac{\partial p}{\partial y}=\frac{1}{\mathrm{Re}}\frac{\partial^2 v}{\partial y^2}$$

شرایط مرزی



دیواره شرقی و غربی:

$x-\text{momentum:}$

$$\cancelto{0}{u}\frac{\partial u}{\partial x}+\cancelto{0}{v}\cancelto{0}{\frac{\partial u}{\partial y}}=-\frac{\partial p}{\partial x}+\frac{1}{\text{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\cancelto{0}{\frac{\partial^2 u}{\partial y^2}}\right)$$

$$\frac{\partial p}{\partial x}=\frac{1}{\text{Re}}\frac{\partial^2 u}{\partial x^2}$$

خروجی روش حجم محدود $\bar{\phi}_{i,j}$

خروجی مورد انتظار $\phi(x,y)$

تقریب مرتبه اول

$$\phi^{(i)}(x) = \phi\Big|_{x=x_i} + O(\Delta x)$$

تقریب مرتبه دوم

$$\phi^{(i)}(x) = \phi\Big|_{x=x_i} + \frac{\partial \phi}{\partial x}\Big|_{x=x_i}(x-x_i) + O(\Delta x^2)$$

تقریب مرتبه سوم

$$\phi^{(i)}(x) = \phi\Big|_{x=x_i} + \frac{\partial \phi}{\partial x}\Big|_{x=x_i}(x-x_i) + \frac{1}{2}\frac{\partial^2 \phi}{\partial x^2}\Big|_{x=x_i}(x-x_i)^2 + O(\Delta x^3)$$

معادله $N-S$

بازسازی جواب حجم محدود

تقریب مرتبه دوم

$$\phi^{(i)}(x) = \phi_i + \left.\frac{\partial \phi}{\partial x}\right|_i (x - x_i) + O(\Delta x^2)$$

$$\bar{\phi}_i \Delta x = \phi_i \Delta x + \int_{x_w}^{x_e} \left.\frac{\partial \phi}{\partial x}\right|_i (x - x_i) dx$$

$$\bar{\phi}_i = \phi_i$$

$$\phi^{(i)}(x) = \bar{\phi}_i + \left.\frac{\partial \phi}{\partial x}\right|_i (x - x_i) + O(\Delta x^2)$$

$$\left.\frac{\partial \phi}{\partial x}\right|_i = \frac{\bar{\phi}_{i+1} - \bar{\phi}_i}{\Delta x} + O(\Delta x)$$

$$\left.\frac{\partial \phi}{\partial x}\right|_i = \frac{\bar{\phi}_i - \bar{\phi}_{i-1}}{\Delta x} + O(\Delta x)$$

$$\left.\frac{\partial \phi}{\partial x}\right|_i = \frac{\bar{\phi}_{i+1} - \bar{\phi}_{i-1}}{2\Delta x} + O(\Delta x^2)$$

بازسازی جواب حجم محدود

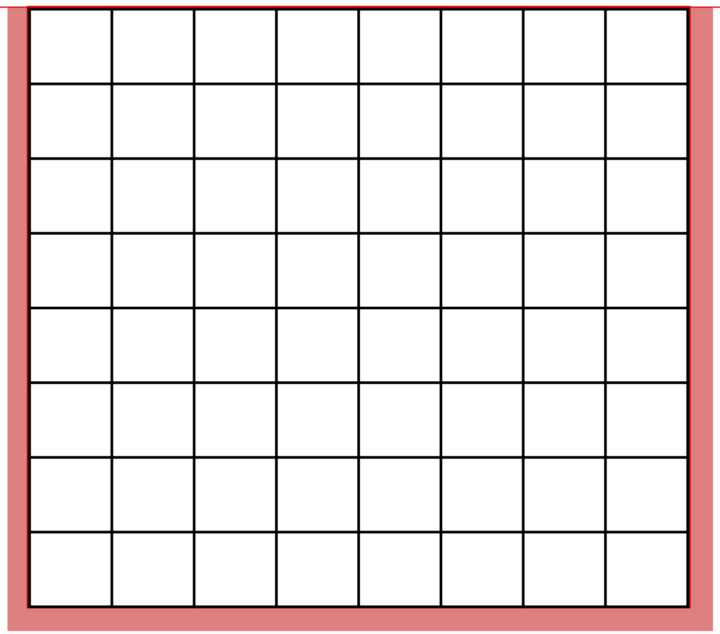
$$\phi^{(i,j)}(x,y)=\bar{\phi}_{i,j}+\left.\frac{\partial\phi}{\partial x}\right|_{i,j}(x-x_{i,j})+\left.\frac{\partial\phi}{\partial y}\right|_{i,j}(y-y_{i,j})+O(\Delta x^2,\Delta y^2)$$

$$u^{(1,j)}(x,y)$$
$$=\bar{u}_{1,j}+\left.\frac{\partial u}{\partial x}\right|_{1,j}(x-x_{1,j})$$
$$+\left.\frac{\partial u}{\partial y}\right|_{1,j}(y-y_{1,j})+O(\Delta x^2,\Delta y^2)$$

$$\frac{\partial^2 u}{\partial x^2}=0$$

$$\frac{\partial p}{\partial v}=\frac{1}{\mathrm{Re}}\frac{\partial^2 v}{\partial y^2}$$

$$u=u_w, v=0$$

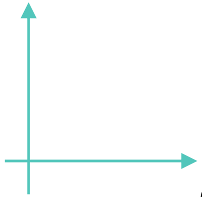


$$u=v=0$$

$$\frac{\partial p}{\partial x}=\frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial x^2}$$

$$u=v=0$$

$$\frac{\partial p}{\partial x}=\frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial x^2}$$



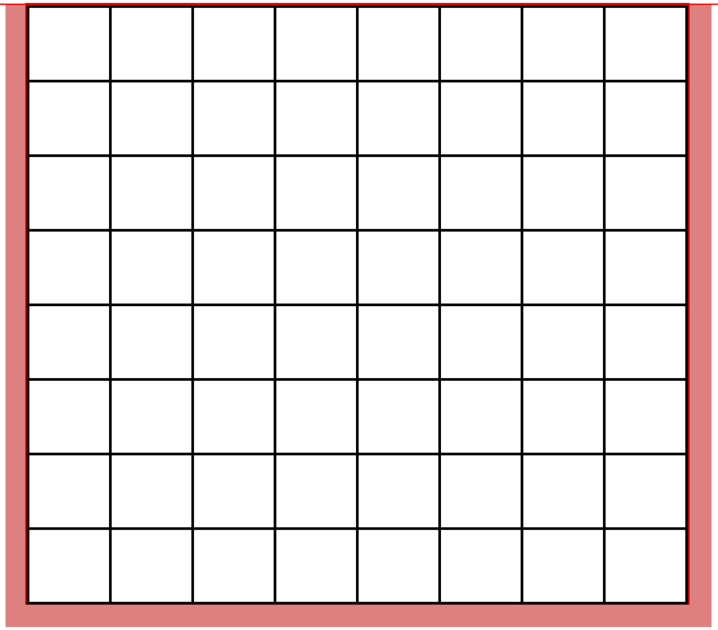
$$u=v=0$$

$$\frac{\partial p}{\partial y}=\frac{1}{\mathrm{Re}}\frac{\partial^2 v}{\partial y^2}$$

بازسازی جواب حجم محدود

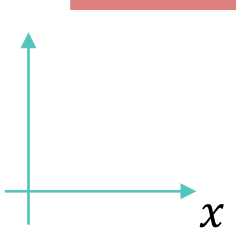
$$\frac{\partial p}{\partial y} = 0$$

$$u = u_w, v = 0$$



$$u = v = 0 \qquad \frac{\partial p}{\partial x} = 0$$

$$u = v = 0 \qquad \frac{\partial p}{\partial x} = 0$$

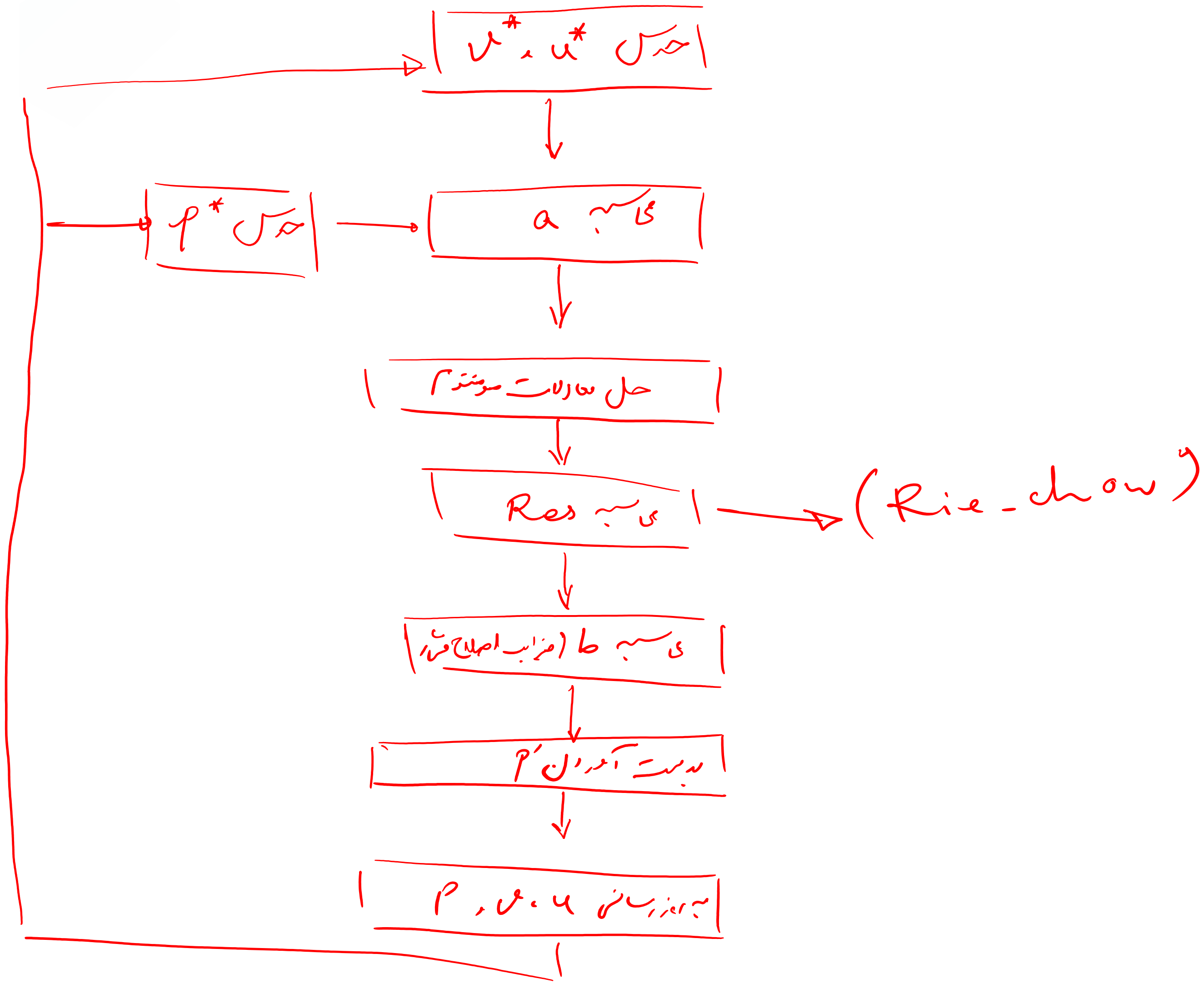


$$u = v = 0$$

$$\frac{\partial p}{\partial y} = 0$$

$$p_{1,1} = 0$$

SIMPLE:


حل u^* و v^*
محاسبه a
حل p^*
حل معادلات مومنتوم
محاسبه Res
(Rie-chow)
محاسبه d (ضرایب اصلاح فشار)
بدست آوردن p'
به‌روزرسانی u و v و p

```
u, v, p = Initialize flow field;
while (||Res||_2 > tol && iteration < Max_iteration)
    update_ghost_cell;  --> ug, vg
    Compute_momentum_equation_coefficients;  --> aE|i,j , aw|i,j , ap , aN , as
    Solve_momentum_equation;  --> u*, v*
    Compute_Residual (||Res||_2)  Rie_chow --> Res
    Compute_pressure_Correction_Eq_Coefficients;  --> b
    Solve_pressure_Correction_Eq;  --> p'
    update_flow_characteristics;  --> (u, v, p)
end while
```

(فقط یک مرحله به روز رسانی گردد)

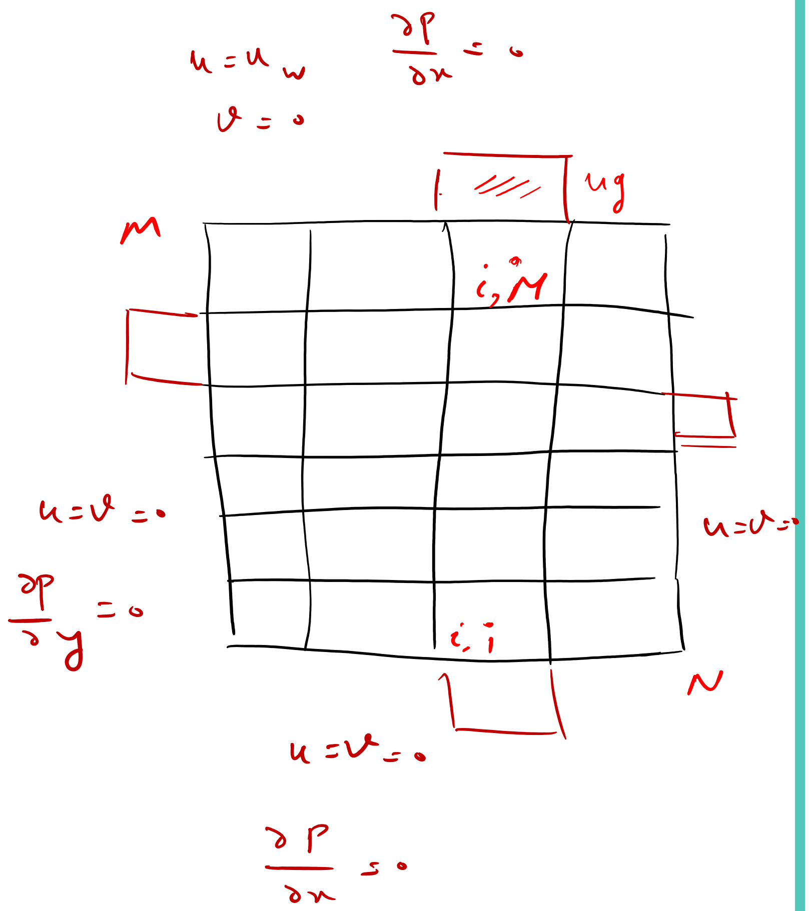
function update_ghost_cell

دیوار بالا :

$$\begin{cases} u_g = 2u_w - u(i,M) \\ v_g = -v(i,M) \\ P_g = P(i,M) \end{cases}$$

دیوار پایین

$$\begin{cases} u_g = -u(i,1) \\ v_g = -v(i,1) \\ P_g = P(i,1) \end{cases}$$



حل مسئله حفره

$[a_P, a_E, a_N, \ldots]$ =

function Compute_momentum_eq_coefficients$(u, v, Re, \Delta x, \Delta y)$

$$a_E(i,j) = \frac{u_e}{\Delta x} - \frac{1}{Re\,\Delta x^2}, \quad u_e = \frac{1}{2}\left(\bar{u}_{i,j} + \bar{u}_{i+1,j}\right)$$

$a_W(i,j)$

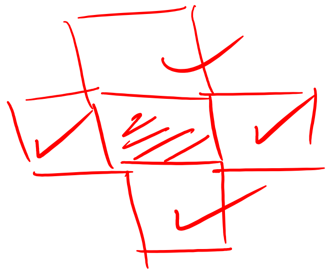
$a_N(i,j)$

$a_S(i,j)$

$a_P(i,j)$

function Solve_momentum_equation

$$a_p u^*_p + \sum a_{nb} u^*_{nb} s - \left(\frac{p^*_{i+1,j} - p^*_{i-1,j}}{2\Delta x} \right)$$



$$u^*_p = \frac{1}{a_p}\left(-\sum a_{nb} u^*_{nb} - \left(\frac{p^*_{i+1,j} - p^*_{i-1,j}}{2\Delta x} \right) \right)$$

فقط یک مرتبه حل کردن

$$u^*_p = ---$$

~~while ($\|\delta u\|_2 < tol$)~~
~~for i=1:N~~
~~for j=1:M~~
~~u^*_p = ...~~
~~end~~
~~end~~

function Compute_Residual

$$Res(i,j) = -\left(\frac{u^*_e - u^*_w}{\Delta x} + \frac{v^*_n - v^*_s}{\Delta y}\right)$$

$$u^*_e = \frac{1}{2}\left(u^*_P + \frac{1}{ap(i,j)}\left(\frac{P^*_E - P^*_W}{2\Delta x}\right) + u^*_E + \frac{1}{ap(i+1,j)}\left(\frac{P^*_{EE} - P^*_P}{2\Delta x}\right) - \frac{1}{a_e}\left(\frac{P^*_E - P^*_P}{\Delta x}\right)\right)$$

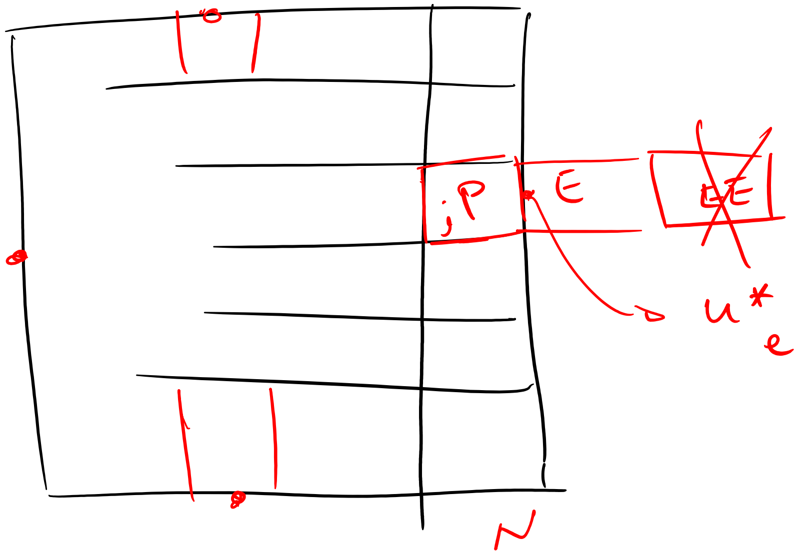
u^*_w

v^*_n (Rie_chow)

v^*_s

$$\frac{1}{a_e} = \frac{1}{2}\left(\frac{1}{ap(i,j)} + \frac{1}{ap(i+1,j)}\right)$$

حل مسئله حفره



$$u^*_e = \frac{1}{2}\left(u(N,j) + u_g\right)$$

function compute_pressure_correction_eq_coeffs

$$b_P P'_P + b_E P'_E + b_W P'_W + b_N P'_N + b_S P'_S = Res$$

$$b_E = -\frac{1}{a_e \Delta x^2} \qquad a_e = \frac{1}{2}\left(ap(i,j) + ap(i+1,j)\right)$$

$$b_W = -\frac{1}{a_w \Delta x^2}$$

$$b_N = -\frac{1}{a_n \Delta y^2}$$

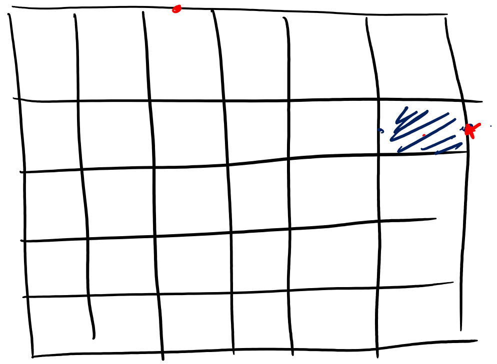
$$b_S = -\frac{1}{a_s \Delta y^2}$$

$$b_P = -(b_E + b_W + b_N + b_S)$$

دیوار شرقی $b_E = 0$

دیوار شمالی $b_N = 0$

$$b_P = -\sum b_{NB}$$



$$u_e = u_{BC}$$

$$u_e = u_e^* + \frac{P'_{i,j} - P'_{i+1,j}}{\Delta x}$$

Function Solve_Pressure_Correction_eq:

$$b_P P'_P + b_E P'_E + b_W P'_W + b_N P'_N + b_S P'_S = Res$$

$$P'_P = \frac{1}{a_P}\left(Res - b_E P'_E - b_W P'_W - b_N P'_N - b_S P'_S\right)$$

```
while (||δp'||_2 > tol && inner_iteration < Max_inner_iteration)
    for i=1:N
        for j=1:M
            P'p = ...
        end
    end
end while
```

10
20
30
40
50

حل مسئله حفره

function update_flow_characteristics

$$p = p^* + \alpha_p p'$$

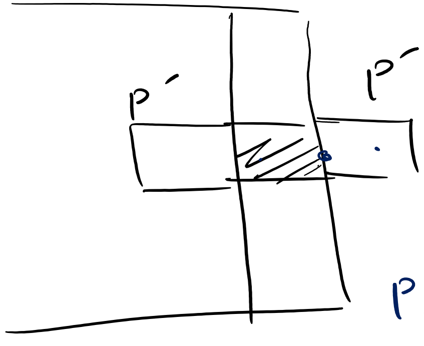
$$u(i,j) = u^*(i,j) - \left(\frac{p'(i+1,j) - p'(i-1,j)}{2\, a_p(i,j)\, \Delta x} \right)$$

$$v(i,j) = v^*(i,j) - \left(\frac{p'(i,j+1) - p'(i,j-1)}{2\, a_p(i,j)\, \Delta y} \right)$$

$$p(i,j) = p(i,j) - p(1,1)$$

$$\left(\frac{\partial p}{\partial n} = 0 \right) \qquad \frac{\partial p^*}{\partial n} = 0 \ \checkmark$$

$$\frac{\partial (p' + p^*)}{\partial n} = 0$$



$$p'_g = p'_{M,j}$$

$$\frac{\partial p'}{\partial n} = 0$$

خسته نباشید